Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability and Statistics **March 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	${}^{38}C_r$ or ${}^{n}C_{34}$	**M1**	Either expression seen OE, no other terms, condone x1
	${}^{38}C_{34}$	**A1**	Correct unsimplified OE
	73815	**A1**	If M0, **SCB1** ${}^{38}C_{34}$ x *k, k* an integer
		3	

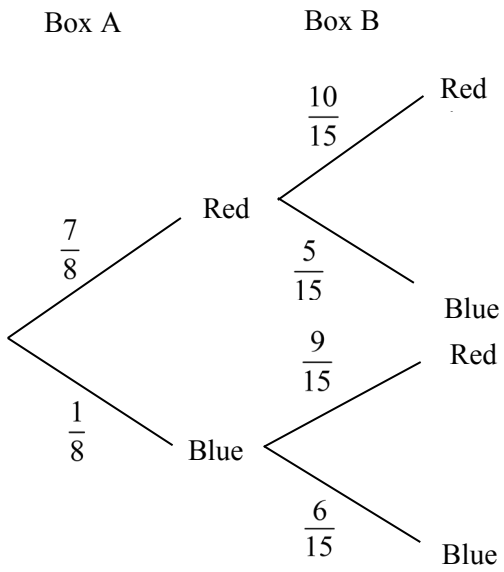
Question	Answer	Marks	Guidance
2(a)	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$	**M1**	One correct term with $0 < p < 1$
	$=\frac{4}{27}+\frac{8}{81}+\frac{16}{243}\left(=\frac{2432}{7776}\right)$	**A1**	Correct expression, accept unsimplified
	$=\frac{76}{243}$ or 0.313	**A1**	
		3	

Question	Answer	Marks	Guidance
2(b)	x | 0 | 1 | 2 | 3 P(x) | $\frac{8}{27}$ | $\frac{12}{27}$ | $\frac{6}{27}$ | $\frac{1}{27}$	**B1**	Probability distribution table with correct values of x, no additional values unless with probability of 0 stated, at least one non-zero probability included
	$P(0) = \left(\frac{2}{3}\right)^3$ $P(1) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \times 3$ $P(2) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2 \times 3$ $P(3) = \left(\frac{1}{3}\right)^3$	**B1** **B1**	1 correct probability seen (may not be in table) **or** 3 or 4 non-zero probabilities summing to 1 All probabilities correct
		3	
2(c)	$E(X) = \left[0 \times \frac{8}{27}\right] + 1 \times \frac{12}{27} + 2 \times \frac{6}{27} + 3 \times \frac{1}{27}$ $= \left[\frac{0}{27}\right] + \frac{12}{27} + \frac{12}{27} + \frac{3}{27}$	**M1**	Correct method from *their* probability distribution table with at least 3 terms, $0 \leqslant$ *their* P(x) $\leqslant 1$, accept unsimplified
	= 1	**A1**	
		2	

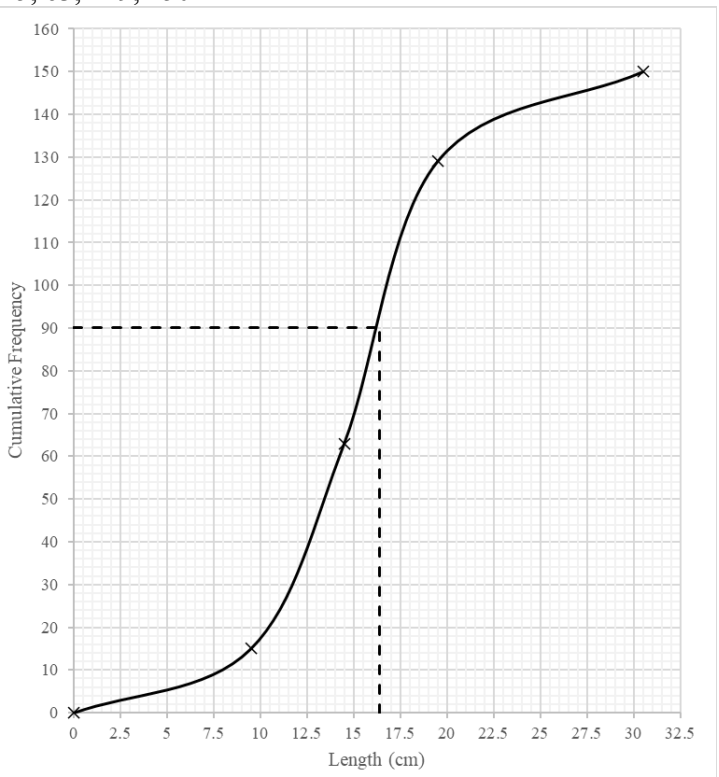
Question	Answer	Marks	Guidance
3(a)	$\mathrm{P}(X > 87) = \mathrm{P}\left(Z > \frac{87-82}{\sigma}\right) = 0.22$	**M1**	Using ± standardisation formula, not σ^2, not $\sqrt{\sigma}$, no continuity correction
	$\mathrm{P}\left(Z < \frac{5}{\sigma}\right) = 0.78$ $\left(\frac{5}{\sigma} =\right) 0.772$	**B1**	AWRT ±0.772 seen B0 for ±0.228
	$\sigma = 6.48$	**A1**	
		3	
3(b)	$\mathrm{P}\left(-\frac{4}{\sigma} < Z < \frac{4}{\sigma}\right) = \mathrm{P}(-0.6176 < Z < 0.6176)$	**M1**	Using ±4 used within a standardisation formula (SOI), allow σ^2, $\sqrt{\sigma}$ and continuity correction
		M1	Standardisation formula applied to **both** *their* ±4
	$\Phi = 0.7317$ Prob = $2\Phi - 1 = 2(0.7317) - 1$	**M1**	Correct area $2\Phi - 1$ oe linked to final solution
	= 0.463	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	R ^ ^ ^ ^ ^ ^ ^ ^ ^ R $\frac{9!}{3!6!}$	**M1**	9! Alone on numerator, $3! \times k$ or $6! \times k$ on denominator
	= 84	**A1**	
		2	
4(b)	^ (B B B) ^ ^ ^ ^ ^	**M1**	$\frac{7!}{6!} \times k$ or $7k$ seen, k an integer > 0
	$\frac{7!}{6!} \times \frac{8 \times 7}{2}$	**M1**	$m \times n(n-1) or\ m \times {}^nC_2 or\ m \times {}^nP_2$, n=7, 8 or 9, m an integer > 0
		M1	n = 8 used in above expression
	= 196	**A1**	
	Alternative for question 4(b)		
	[Arrangements, blues together – Arrangements with blues together and reds together =] $\frac{9!}{2!6!} - \frac{8!}{6!}$	**M1**	9! Seen alone or as numerator with subtraction
	= [252 – 56]	**M1**	8! Seen alone or as numerator in a second term and no other terms
		M1	All terms divided by 6! x k, k an integer
	= 196	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	$1 - P(6, 7, 8)$ $= 1 - ({}^8C_6\ 0.7^6 0.3^2 + {}^8C_7\ 0.7^7 0.3^1 + 0.7^8)$	**M1**	One term ${}^8C_x\ p^x (1-p)^{8-x}$, $0 < p < 1,\ x \neq 0$
	$= 1 - 0.55177$	**A1**	Correct unsimplified expression, or better
	$= 0.448$	**A1**	
	Alternative method for question 5(a)		
	$P(0, 1, 2, 3, 4, 5)$ $= 0.3^8 + {}^8C_1\ 0.7^1 0.3^7 + {}^8C_2\ 0.7^2 0.3^6 + {}^8C_3\ 0.7^3 0.3^5 +$ ${}^8C_4 0.7^4 0.3^4 + {}^8C_5\ 0.7^5 0.3^3$	**M1**	One term ${}^8C_x\ p^x (1-p)^{8-x}$, $0 < p < 1,\ x \neq 0$
		A1	Correct unsimplified expression, or better
	$= 0.448$	**A1**	
		3	
5(b)	Mean $= 120 \times 0.7 = 84$ Var $= 120 \times 0.7 \times 0.3 = 25.2$	**B1**	Correct mean and variance, allow unsimplified
	$P(\text{ more than } 75) = P\left(z > \frac{75.5 - 84}{\sqrt{25.2}}\right)$	**M1**	Substituting *their* μ and σ into the ±standardising formula (any number), not σ^2, not $\sqrt{\sigma}$
		M1	Using continuity correction 75.5 or 74.5
	$P(z > -1.693)$	**M1**	Appropriate area Φ, from final process, must be a probability
	$= 0.955$	**A1**	Allow $0.9545 < p \leqslant 0.955$
		5	

Question	Answer	Marks	Guidance
6(a)		**B1**	Both correct probs, box A
		B1	2 probs correct for box B
		B1	All correct probs for box B
		3	
6(b)	$\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{9}{15}$	**M1**	Two 2 factor terms added, correct or FT *their* **6(a)**.
	$=\frac{44}{120}\left[\frac{11}{30}\text{ or }0.367\right]$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
6(c)	$\text{P(A blue } \vert \text{B blue)} = \frac{\text{P(A blue} \cap \text{ B blue)}}{\text{P(B blue)}}$ $= \frac{\frac{1}{8}\times\frac{6}{15}}{\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}} = \frac{\frac{1}{20}}{\frac{41}{120}}$	**M1**	*their* $\frac{1}{8}\times\frac{6}{15}$ seen as numerator or denom of fraction
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen as denominator
	$= \frac{6}{41}$ or 0.146	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	15, 63, 129, 150  	**B1**	Correct cumulative frequencies seen (may be on graph)
		B1	0 ⩽ Horizontal axis ⩽ 30, 0 ⩽ vertical axis ⩽ 150 Labels correct: length cm, cf
		M1	At least 3 points plotted at upper end points (e.g. allow 9, 9.5, 10) with a linear horizontal scale.
		A1	Linear vertical scale, all points at correct upper end points (9.5 etc.), curve drawn accurately, joined to (0,0) (condone (–0.5, 0))
		4	
7(b)	60% of 150 = 90	**M1**	90 seen or implied by use on graph
	Approx. 16.5 [cm]	**A1FT**	FT *their* increasing cumulative frequency graph, Use of graph must be seen. If no clear evidence of use of graph **SCB1FT** correct value from *their* graph
		2	

Question	Answer	Marks	Guidance
7(c)	Midpoints: 4.75, 12, 17, 25	**M1**	At least 3 correct midpoints used (39449.4375 implies M1)
	Var = $\dfrac{4.75^2 \times 15 + 12^2 \times 48 + 17^2 \times 66 + 25^2 \times 21}{150} - 15.295^2$	**M1**	Using midpoints ±0.5 in correct var formula, including subtraction of *their* μ^2.
	= 29.1	**A1**	
		3	

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Mathematics Specific Marking Principles	
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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

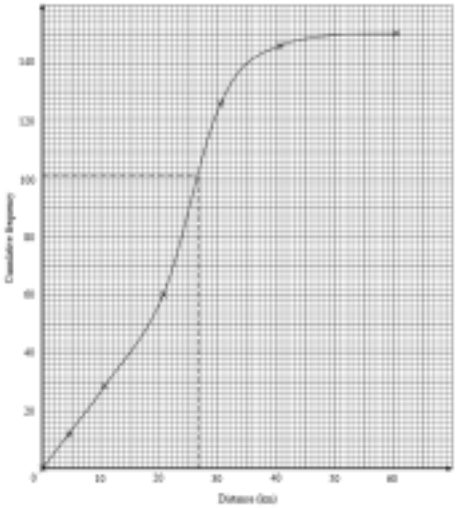
Question	Answer	Marks	Guidance
1(a)	$\left[\left(\frac{4}{5}\right)^7\frac{1}{5}=\right]\ \frac{16384}{390625}$ or 0·0419[43…]	**B1**	Evaluated, final answer.
		1	
1(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2+pq^3+ pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$ Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5+{}^5\text{C}_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)+{}^5\text{C}_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2+{}^5\text{C}_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3+{}^5\text{C}_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	**M1**	$(p)^5+{}^5\text{C}_4(p)^4(q)+{}^5\text{C}_3(p)^3(q)^2+{}^5\text{C}_2(p)^2(q)^3+{}^5\text{C}_1(p)(q)^4$ or $(p)^6+{}^6\text{C}_5(p)^5(q)+{}^6\text{C}_4(p)^4(q)^2+{}^6\text{C}_3(p)^3(q)^3$ $+{}^6\text{C}_2(p)^2(q)^4+{}^6\text{C}_1(p)(q)^5,\ 0 < p < 1, p + q = 1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
		2	

Question	Answer	Marks	Guidance
2(a)	$0.2[\times 1]+0.45\times 0.4+0.35\times 0.3$	**M1**	0·2 [× 1] + 0·45 × *b* + 0·35 × *c*, *b* = 0·4, 0·6 *c* = 0·3, 0·7
	0·485 or $\frac{97}{200}$	**A1**	
		2	
2(b)	$P(Y \mid \overline{H}) = \frac{P(Y \cap \overline{H})}{P(\overline{H})} = \frac{0.35\times 0.7}{1-\textit{their}\textbf{(a)}} = \frac{0.245}{0.515}$	**B1**	0·35 × 0·7 or 0·245 seen as numerator or denominator of fraction.
		M1	0·515 or 1 – *their* **(a)** or [0·3 × 0 +] 0·45 × *d* + 0·35 × *e*, where *d = their b′, e = their c′* seen as denominator of fraction.
	0·476 or $\frac{49}{103}$	**A1**	0·4757 ⩽ *p* ⩽ 0·476
		3	

Question	Answer	Marks	Guidance
3(a)	$\mathrm{P}\left(\left(\frac{85-96}{18}\right) < z < \left(\frac{100-96}{18}\right)\right)$	**M1**	Use of ±standardisation formula once with appropriate values substituted, no continuity correction, not σ^2 or $\sqrt{\sigma}$.
	$\mathrm{P}(-0.6111 < z < 0.2222)$ $= \Phi(0.2222) + \Phi(0.6111) - 1$ $= 0.5879 + 0.7294 - 1$	**M1**	Appropriate area Φ, from final process, must be probability. Use of (1 – *z*) implies M0.
	0·317	**A1**	Final answer which rounds to 0·317.
		3	

Question	Answer	Marks	Guidance
3(b)	$z = \pm 1{\cdot}175$	**B1**	$1{\cdot}17 \leqslant z \leqslant 1{\cdot}18$ or $-1{\cdot}18 \leqslant z \leqslant -1{\cdot}17$
	$-1.175 = \dfrac{t-96}{18}$	**M1**	An equation using ±standardisation formula with a z-value, condone σ^2, $\sqrt{\sigma}$ or continuity correction. E.g. equating to 0·88, 0·12, 0·8106, 0·1894, 0·5478, 0·4522, ±0·175 or ±2·175 implies M0.
	74·85 or 74·9	**A1**	$74{\cdot}85 \leqslant t \leqslant 74{\cdot}9$
		3	

Question	Answer	Marks	Guidance
4(a)	x | 1 | 2 | 3 | 4 prob | $4k$ | $6k$ | $6k$ | $4k$	**B1**	Table with × values and one correct probability expressed in terms of k. Condone any additional × values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.
		2	
4(b)	$[4k + 6k + 6k + 4k = 1]\ k = \frac{1}{20}\ \ (= 0.05)$	**B1**	Correct value for k SOI. May be calculated in **4(a)**. **SC B1** If denominator $20k$ used throughout.
	$E(X) = 1\times\frac{4}{20} + 2\times\frac{6}{20} + 3\times\frac{6}{20} + 4\times\frac{4}{20} = \frac{4}{20} + \frac{12}{20} + \frac{18}{20} + \frac{16}{20}$ $(= 2.5)$	**M1**	Accept unsimplified expression. Condone $4k + 12k + 18k + 16k$. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator $20k$.
	$\text{Var}(X) = 1^2\times\frac{4}{20} + 2^2\times\frac{6}{20} + 3^2\times\frac{6}{20} + 4^2\times\frac{4}{20} - \left(their\ 2\frac{1}{2}\right)^2$ $= (4 + 24 + 54 + 64)\times their\,0.05 - (their\,2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20} + (2-2.5)^2\times\frac{6}{20} + (3-2.5)^2\times\frac{6}{20} + (4-2.5)^2\times\frac{4}{20}$	**M1**	Appropriate variance formula with *their* numerical probabilities using *their* $(\text{E}(X))^2$, accept unsimplified, with *their* k substituted. **Special ruling:** If denominator $20k$ used throughout, accept appropriate variance formula in terms of k.
	1·05	**A1**	AG, NFWW.
		4	

Question	Answer	Marks	Guidance
5(a)	<table><tr><td>Distance</td><td>0-4</td><td>5-10</td><td>11-20</td><td>21-30</td><td>31-40</td><td>41-60</td></tr><tr><td>Upper boundary</td><td>4·5</td><td>10·5</td><td>20·5</td><td>30·5</td><td>40·5</td><td>60·5</td></tr><tr><td>Cumulative frequency</td><td>12</td><td>28</td><td>60</td><td>126</td><td>146</td><td>150</td></tr></table>	**B1**	Correct cumulative frequencies seen (may be by table or plotted accurately on graph), condone 12 not stated.
		**B1**	Axes labelled 'distance (or d) [in] km' from 0 to 60 **and** 'cumulative frequency' (or cf) from 0 to 150.
		M1	At least 5 points plotted at upper end points for d (allow upper boundary ±0·5) with a linear scale for distance, condone 0 – 4 interval inaccurate, no scale break on axis. Not bar graph/histogram unless clear indication of upper end point only of each bar.
		A1	All plotted correctly at correct upper end points (4.5 etc.) with both scales linear ($0 \leqslant d \leqslant 60$, $0 \leqslant \text{cf} \leqslant 150$), curve drawn accurately joined to (0,0), cf line>150, no daylight if >150.
		4	
5(b)	70% of 150 = 105	**M1**	105 seen or implied by indication on grid.
	Approx. 27	**A1 FT**	Strict FT *their* increasing cumulative frequency graph, use of graph must be seen. If no clear evidence of use of graph: **SC B1 FT** correct value from *their* increasing cumulative frequency graph.
		2	

Question	Answer	Marks	Guidance
5(c)	Midpoints: 2.25, 7.5, 15.5, 25.5, 35.5, 50.5	**B1**	At least 5 correct midpoints seen.
	$\text{Mean} = \dfrac{2.25\times12+7.5\times16+15.5\times32+25.5\times66+35.5\times20+50.5\times4}{150}$ $= \dfrac{27+120+496+1683+710+202}{150}$	**M1**	Using 6 midpoint attempts (e.g. 2·25 ±0·5), condone one error not omission, multiplied by frequency, accept unevaluated, denominator either correct or *their* Σ frequencies.
	$\left[=\dfrac{3238}{150}\right] = 21.6,\ 21\dfrac{44}{75}$	**A1**	Evaluated, WWW, accept 21·5[866…].
		3	

Question	Answer	Marks	Guidance
6(a)	$\dfrac{11!}{2!2!2!}$	**M1**	11! alone as numerator. 2! × *m*! × *n*! on denominator, *m* = 1, 2, *n* = 1, 2. no additional terms, no additional operations.
	4989600	**A1**	Exact answer only.
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$ Number of ways of placing As in non-adjacent places = ${}^8C_2$ $\frac{7!}{2!}\times {}^8C_2$	**B1**	$\frac{7!}{2!}\times k$ seen, k an integer > 1.
		M1	$m\times n(n-1)$ or $m\times {}^nC_2$ or $m\times {}^nP_2$, n = 7, 8 or 9, m an integer > 1.
		M1	$\frac{7!}{p!}\times {}^8C_2$ or $\frac{7!}{p!}\times {}^8P_2$, p integer $\geqslant$ 1, condone 2520×28.
	= 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end = $\frac{9!}{2!2!}$ Arrangements with R at ends and As together = $\frac{8!}{2!}$ With As not together = $\frac{9!}{2!2!}-\frac{8!}{2!}$	**M1**	$\frac{9!}{2!m!}-k$, 90720 > k integer > 1, m = 1, 2.
		B1	$s-\frac{8!}{2!}$, s an integer >1
		M1	$\frac{9!}{p}-\frac{8!}{q}$, p, q integers $\geqslant$ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	R R A L _ _ $^5C_2$ = 10 R R A L L _ $^5C_1$ = 5 R R A A L _ $^5C_1$ = 5 R R A A L L = 1	**M1**	^5C_x seen alone or $^5C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0 < x < 5$ linked to an appropriate scenario.
		A1	$^5C_2 \times k$, $k = 1$ oe or $^5C_1 \times m$, $m = 1,2$ oe alone. **SC** if ^5C_x not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. $^2C_w \times {}^2C_x \times {}^2C_y \times {}^5C_z$, $w+x+y+z=6$ identifies w Rs, × As and y Ls.
	[Total =] 21	**A1**	WWW, only dependent on 2nd M mark. Note: $^5C_2 + {}^5C_1 + {}^5C_1 + 1 = 21$ is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = $^7C_2$	**M1**	^7C_x seen alone or $^7C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$. Condone ^7P_x or $^7P_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$.
		M1	$^7C_2 \times k$, $2 \geqslant k \geqslant 1$oe
		A1	$^7C_2 \times k$, $k = 1$oe no other terms.
	[Total =] 21	**A1**	Value stated.
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\left[\frac{104+31}{400}=\right]\frac{135}{400}, \frac{27}{80}, 0{\cdot}3375$	**B1**	Evaluated, exact value.
		1	
7(a)(ii)	**Method 1**		
	$\mathrm{P}(M)=\frac{180}{400}, 0{\cdot}45$ $\mathrm{P}(S)=\frac{135}{400}, 0{\cdot}3375$ $\mathrm{P}(M\cap S)=\frac{31}{400}, 0{\cdot}0775$ $\frac{180}{400}\times\frac{135}{400}=\frac{243}{1600}, 0.151875\neq\frac{31}{400}$ so NOT independent	**M1**	*Their* P(M) × *their* P(S) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
	Method 2		
	$\mathrm{P}(M\cap S)=\frac{31}{400}$ $\mathrm{P}(S)=\frac{135}{400}$ $\mathrm{P}(M)=\frac{180}{400}$ $\mathrm{P}(M\|S)=\frac{\frac{31}{400}}{\frac{135}{400}}=\frac{31}{135}, 0.2296\ldots\neq\frac{180}{400}$ so NOT independent	**M1**	$[\mathrm{P}(M\|S)=]\frac{\textit{their}\,\mathrm{P}(M\cap S)}{\textit{their}\,\mathrm{P}(S)}$ (oe) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
		2	

Question	Answer	Marks	Guidance
7(b)(i)	**Method 1** [1 – P(0,1,2)]		
	$= 1 - ({}^{10}C_0\, 0{\cdot}3^0\, 0{\cdot}7^{10} + {}^{10}C_1\, 0{\cdot}3^1\, 0{\cdot}7^9 + {}^{10}C_2\, 0{\cdot}3^2\, 0{\cdot}7^8)$	**M1**	${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < \times < 10$, $0 < p < 1$, any p.
	$= 1 - (0{\cdot}028248 + 0{\cdot}121061 + 0{\cdot}233474)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket, condone recovery from poor notation.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
	Method 2 [P(3,4,5,6,7,8,9,10) =]		
	${}^{10}C_3\, 0{\cdot}3^3\, 0{\cdot}7^7 + {}^{10}C_4\, 0{\cdot}3^4\, 0{\cdot}7^6 + {}^{10}C_5\, 0{\cdot}3^5\, 0{\cdot}7^5 + {}^{10}C_6\, 0{\cdot}3^6\, 0{\cdot}7^4 + {}^{10}C_7\, 0{\cdot}3^7\, 0{\cdot}7^3 + {}^{10}C_8\, 0{\cdot}3^8\, 0{\cdot}7^2 + {}^{10}C_9\, 0{\cdot}3^9\, 0{\cdot}7^1 + {}^{10}C_{10}\, 0{\cdot}3^{10}\, 0{\cdot}7^0$	**M1**	${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < \times < 10$, $0 < p < 1$, any p.
		A1	Correct unsimplified expression.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	[p = 0·3] Mean = 0·3 × 90 = 27; variance = 0·3 × 90 × 0·7 = 18·9	**B1**	Correct mean and variance, allow unsimplified. Condone $\sigma = 4{\cdot}347$ evaluated.
	$\mathrm{P}(X < 32) = P\left(z < \frac{31.5-27}{\sqrt{18.9}}\right)$	**M1**	Substituting *their* μ and σ (not σ^2, $\sqrt{\sigma}$) into the ±standardising formula with a numerical value for '31·5'.
		M1	Using either 31·5 or 32·5 within a ±standardising formula with numerical values for *their* μ and σ (condone σ^2, $\sqrt{\sigma}$).
	$= \Phi(1.035)$	**M1**	Appropriate area Φ, from standardisation formula P(z<…) in final solution, must be probability.
	= 0·850	**A1**	Allow $0{\cdot}8495 < p \leqslant 0{\cdot}85(0)$, final answer WWW.
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

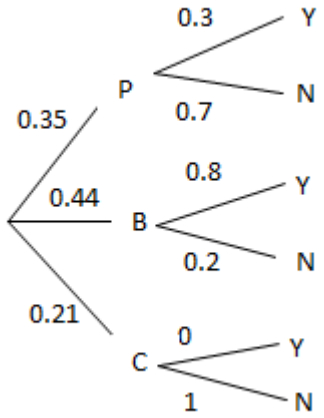
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)	Prob of 4 (from 1,3, 3,1 or 2,2) = $\frac{3}{36}=\frac{1}{12}$ **AG**	**B1**
		1
1(b)	Mean = $\frac{1}{\frac{1}{12}}=12$	**B1**
		1
1(c)	$\left(\frac{11}{12}\right)^5\times\frac{1}{12}=0.0539$ or $\frac{161051}{2985984}$	**B1**
		1
1(d)	$1-\left(\frac{11}{12}\right)^7$	**M1**
	0.456 or $\frac{16344637}{35831808}$	**A1**
		2

Question	Answer	Marks
2(a)	6!	**M1**
	720	**A1**
		2
2(b)	Total number: $\frac{9!}{3!2!}(30240)$	**M1**
	Number with Ls together $= \frac{8!}{3!}(6720)$	**M1**
	Number with Ls not together $= \frac{9!}{3!2!} - \frac{8!}{3!}$ $= 30\,240 - 6720$	**M1**
	23 520	**A1**
	Alternative method for question 2(b)	
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	
	$7! \times k$ in numerator, k integer ≥ 1	**M1**
	$8 \times 7 \times m$ in numerator or $8C2 \times m$, m integer ≥ 1	**M1**
	3! in denominator	**M1**
	23 520	**A1**
		4

Question	Answer	Marks
3(a)	x: 0 | 1 | 2 | 3 Probability: $\frac{1}{56}$ | $\frac{15}{56}$ | $\frac{30}{56}$ | $\frac{10}{56}$ (**B1** for probability distribution table with correct outcome values)	**B1**
	$P(0) = \frac{3}{8}\times\frac{2}{7}\times\frac{1}{6} = \frac{1}{56}$ $P(1) = \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3 = \frac{15}{56}$ $P(2) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times 3 = \frac{30}{56}$ $P(3) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6} = \frac{10}{56}$ (**M1** for denominator $8\times7\times6$)	**M1**
	Any one probability correct (with correct outcome)	**A1**
	All probabilities correct	**A1**
		4
3(b)	$1 - P(8, 9, 10) = 1 - \left[{}^{10}C_8\, 0.64^8 0.36^2 + {}^{10}C_9\, 0.64^9 0.36^1 + 0.64^{10} \right]$	**M1**
	$1 - (0.164156 + 0.064852 + 0.11529)$	**M1**
	0.759	**A1**
		3

Question	Answer	Marks
4	Scenarios: 2P 3V 2G $^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (**M1** for $^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	**M1**
	Two unsimplified products correct	**B1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total: 13 720	**A1**
		4

Question	Answer	Marks
5(a)		
	Fully correct labelled tree for method of transport with correct probabilities.	**B1**
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0	**B1**
		2
5(b)	$0.35 \times 0.3 + 0.44 \times 0.8 \, (+ 0)$	**M1**
	0.457	**A1**
		2

Question	Answer	Marks
5(c)	P(not B|not fruit) = $\dfrac{\mathrm{P}(\mathrm{B}'\cap\mathrm{F}')}{\mathrm{P}(\mathrm{F}')}$	**M1**
	$\dfrac{0.35\times0.7+0.21\times1}{1-\textit{their}(\mathbf{b})}$	**M1**
	$\dfrac{0.455}{0.543}$ (**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator)	**M1**
	0.838 or $\dfrac{455}{543}$	**A1**
		4

Question	Answer	Marks
6(a)	$\mathrm{P}\left(\dfrac{50-54}{6.1}<z<\dfrac{60-54}{6.1}\right)=\mathrm{P}(-0.6557<Z<0.9836)$	**M1**
	Both values correct	**A1**
	$\Phi(0.9836)-\Phi(-0.6557)=\Phi(0.9836)+\Phi(0.6557)-1$ $=0.8375+0.7441-1$ (Correct area)	**M1**
	0.582	**A1**
		4

Question	Answer	Marks
6(b)	$\frac{45-\mu}{\sigma} = -0.994$	**B1**
	$\frac{56-\mu}{\sigma} = 1.372$	**B1**
	One appropriate standardisation equation with μ, σ, z-value (not probability) and 45 or 56.	**M1**
	$11 = 2.366\,\sigma$ (**M1** for correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable)	**M1**
	$\sigma = 4.65$, $\mu = 49.6$	**A1**
		5

Question	Answer	Marks
7(a)	Class widths: 10, 5, 15, 20, 10	**M1**
	Frequency density = frequency/*their* class width: 1.8, 4.8, 2, 1, 0.8	**M1**
	All heights correct on diagram (using a linear scale)	**A1**
	Correct bar ends	**B1**
	Bar ends: 10.5, 15.5, 30.5, 50.5, 60.5	**B1**
		5
7(b)	11 – 15 and 31 – 50	**B1**
	Greatest IQR = 50 – 11 = 39	**B1**
		2
7(c)	$\text{Mean} = \frac{18\times5.5+24\times13+30\times23+20\times40.5+8\times55.5}{100} = \frac{2355}{100} = 23.6$	**B1**
	$\text{Var} = \frac{18\times5.5^2+24\times13^2+30\times23^2+20\times40.5^2+8\times55.5^2}{100} - \text{mean}^2$	**M1**
	$\frac{77917.5}{100} - \text{mean}^2 = 224.57$	**A1**
	Standard deviation = 15.0 (**FT** *their* variance)	**A1 FT**
		4

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	$\sum x - 50n = 144$	**B1**
	$50n + 144 = 944$	**M1**
	$n = 16$	**A1**
		3

Question	Answer	Marks
2(a)	$\frac{56}{500}$ or $\frac{14}{125}$ or 0.112	**B1**
		1
2(b)	$\mathrm{P}(\mathrm{D}\mid\mathrm{S}) = \frac{\mathrm{P}(\mathrm{D}\cap\mathrm{S})}{\mathrm{P}(\mathrm{S})} = \frac{120}{280}$	**M1**
	$\frac{120}{280}$ or $\frac{3}{7}$	**A1**
		2

Question	Answer	Marks
2(c)	$P(\text{hockey}) = \frac{220}{500} = 0.44$ $P(\text{Amos or Benn}) = \frac{242}{500} = 0.484$ $P(\text{hockey} \cap \text{A or B}) = \frac{104}{500} = 0.208$ $P(H) \times P(A \cup B) = P(H \cap (A \cup B))$ if independent	**M1**
	$\frac{220}{500} \times \frac{242}{500} = \frac{1331}{6250}$ so not independent	**A1**
		2

Question	*Answer*	Marks
3(a)	Median = 0.238	**B1**
	UQ = 0.245, LQ = 0.231, So IQR = 0.245 – 0.231	**M1**
	0.014	**A1**
		3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td rowspan="5">3(b)</td><td>
<table>
<tr><td></td><td></td><td>LQ</td><td>M</td><td>UQ</td><td></td></tr>
<tr><td>A</td><td>0.220</td><td>0.231
FT</td><td>0.238
FT</td><td>0.245
FT</td><td>0.254</td></tr>
<tr><td>B</td><td>0.211</td><td>0.224</td><td>0.232</td><td>0.243</td><td>0.256</td></tr>
</table>
</td><td></td></tr>
<tr><td>Medians and quartiles correctly plotted for A or B</td><td>B1</td></tr>
<tr><td>End points correct for A or B</td><td>B1</td></tr>
<tr><td>Completely correct, including scale</td><td>B1</td></tr>
<tr><td></td><td>3</td></tr>
<tr><td rowspan="3">3(c)</td><td>Lengths of rods produced by machine A are longer.
(B1 for comparison of central tendency)</td><td>B1</td></tr>
<tr><td>Lengths of rods produced by machine A are less spread out
(B1 for comparison of spread)</td><td>B1</td></tr>
<tr><td></td><td>2</td></tr>
</table>

Question	Answer	Marks
4(a)	$\mathrm{P}(X < 25) = \mathrm{P}\left(z < \frac{25-40}{12}\right) = \mathrm{P}(z < -1.25)$	**M1**
	$1 - 0.8944$	**M1**
	0.106	**A1**
		3
4(b)	0.8944 divided by 3 (**M1** for 1 – *their* **(a)** divided by 3)	**M1**
	0.298 **AG**	**A1**
		2
4(c)	0.2981 gives $z = 0.53$	**B1**
	$\frac{h-40}{12} = 0.53$	**M1**
	$h = 46.4$	**A1**
		3

Question	Answer	Marks
5(a)	<table><tr><td></td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr><tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr></table>	**M1**
	$\frac{7}{15}$ **AG**	**A1**
		2
5(b)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>$\frac{2}{15}$</td><td>$\frac{6}{15}$</td><td>$\frac{7}{15}$</td></tr></table>	**B1**
	P(1) or P(2) correct	**B1**
	3rd probability correct, **FT** sum to 1	**B1**
		3

Question	Answer	Marks
5(c)	$\mathrm{E}(X)=\frac{2+12+21}{15}=\frac{35}{15}=\frac{7}{3}$	**B1**
	$\mathrm{Var}(X)=\frac{1^2\times2+2^2\times6+3^2\times7}{15}-\left(\frac{7}{3}\right)^2$	**M1**
	$\frac{22}{45}(0.489)$	**A1**
		3

Question	*Answer*	Marks
6(a)	$\frac{8!}{3!}$	**M1**
	6720	**A1**
		2

Question	Answer	Marks
6(b)	Total number = $\frac{10!}{2!3!}$ (302400) (A)	**B1**
	With Es together = $\frac{9!}{3!}$ (60480) (B)	**B1**
	Es not together = *their* (A) – *their* (B)	**M1**
	241920	**A1**
	Alternative method for question 6(b)	
	_ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ $\frac{8!}{3!} \times \frac{9 \times 8}{2}$	
	8! × *k* in numerator, *k* integer ⩾ 1, denominator ⩾ 1	**B1**
	3! × *m* in denominator, *m* integer ⩾ 1	**B1**
	Their $\frac{8!}{3!}$ Multiplied by ${}^{9}C_{2}$ (OE) only (no additional terms)	**M1**
	241920	**A1**
		4

Question	Answer	Marks
6(c)	Scenarios: E M M M $^5C_0 = 1$ E M M _ $^5C_1 = 5$ E M _ _ $^5C_2 = 10$	**M1**
	Summing the number of ways for 2 or 3 correct scenarios	**M1**
	Total = 16	**A1**
		3

Question	Answer	Marks
7(a)	$1 - P(10, 11, 12)$ $= 1 - [^{12}C_{10}\,0.72^{10}0.28^2 + {}^{12}C_{11}\,0.72^{11}0.28^1 + 0.72^{12}]$	**M1**
	$1 - (0.19372 + 0.09057 + 0.01941)$	**A1**
	0.696	**A1**
		3
7(b)	$0.28^3 \times 0.72 = 0.0158$	**B1**
		1

Question	Answer	Marks
7(c)	Mean = $100 \times 0.72 = 72$ Var = $100 \times 0.72 \times 0.28 = 20.16$	**M1**
	P(less than 64) = $\mathrm{P}\left(z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$ (**M1** for substituting *their* μ *and* σ into ±standardisation formula with a numerical value for '*63.5*')	**M1**
	Using either 63.5 or 64.5 within a ±standardisation formula	**M1**
	Appropriate area Φ, from standardisation formula P(z<…) in final solution = P($z < -1.893$)	**M1**
	0.0292	**A1**
		5

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

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- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
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GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
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Types of mark

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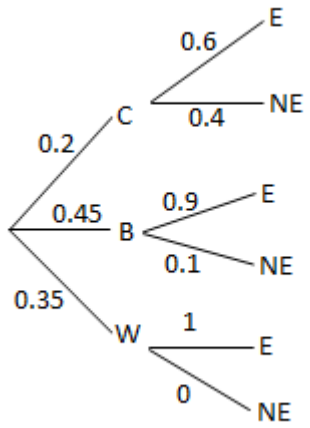
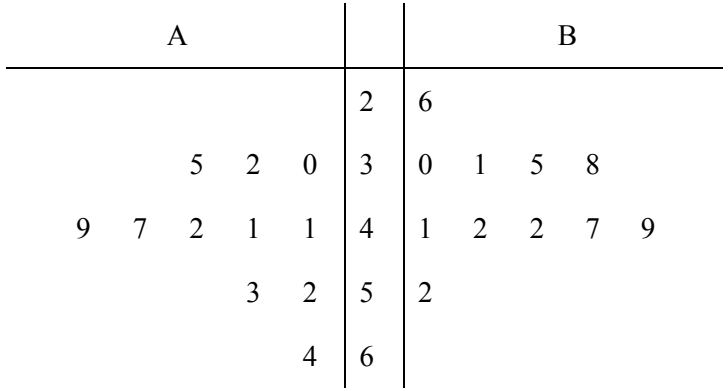
Question	Answer	Marks
1(a)		
	Fully correct labelled tree for method of transport with correct probabilities.	**B1**
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the probability 0.	**B1**
		2
1(b)	$P(C\|E) = \frac{P(C \cap E)}{P(E)} = \frac{0.2\times0.6}{0.2\times0.6+0.45\times0.1+0.35\times1}$	**M1**
	Summing three appropriate 2-factor probabilities	**M1**
	$\frac{0.12}{0.515}$	**A1**
	0.233 or $\frac{12}{515}$	**A1**
		4

Question	Answer	Marks
2(a)	$0.22^3 = 0.0106$	**B1**
		1
2(b)	$P(2, 3, 4) = {}^{16}C_2\ 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$	**M1**
	$0.179205 + 0.235877 + 0.216221$	**A1**
	0.631	**A1**
		3

Question	Answer	Marks
3(a)	$P(X < 21) = P\left(z < \frac{21 - 15.8}{4.2}\right) = \Phi(1.238)$	**M1**
	0.892	**A1**
		2
3(b)	$z = \pm 0.674$	**B1**
	$\frac{k - 15.8}{4.2} = 0.674$	**M1**
	18.6	**A1**
		3

Question	Answer	Marks
4(a)	<table><tr><td>–1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>2</td></tr><tr><td>2</td><td>3</td><td>3</td><td>4</td></tr></table> <table><tr><td>x</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr></table> Probability distribution table with correct scores with at least one probability	**B1**
	At least 4 probabilities correct	**B1**
	All probabilities correct	**B1**
		3
4(b)	$\text{E(X)} = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$	**B1**
	$\text{Var(X)} = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$	**M1**
	$\frac{37}{18}$ $(= 2.06)$	**A1**
		3

Question	Answer	Marks
5(a)	$\frac{1}{\frac{1}{4}} = 4$	**B1**
		1
5(b)	$\frac{9}{64}$ $\left(= 0.141\right)$	**B1**
		1
5(c)	$P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$ (**FT** *their* probability/mean from part **(a)**)	**M1**
	0.763	**A1**
		2
5(d)	Mean $= 80 \times 0.25 = 20$ Var $= 80 \times 0.25 \times 0.75 = 15$	**M1**
	$P(\text{more than } 25) = P\left(z > \frac{25.5 - 20}{\sqrt{15}}\right)$	**M1**
	$P(z > 1.42)$	**M1**
	$1 - 0.9222$	**M1**
	0.0778	**A1**
		5

Question	Answer	Marks
6(a)	 KEY 1 | 4 | 2 means \$41 000 for A and \$42 000 for B Correct stem	**B1**
	Correct A on LHS	**B1**
	Correct B on same diagram	**B1**
	Correct key for *their* diagram, both companies identified and correct units	**B1**
		4
6(b)	Median = [\$]42 000	**B1**
	LQ = [\$]35 000 UQ = [\$]52 000	**B1**
	IQR = [\$]17 000 (**FT** if $49000 \leqslant UQ \leqslant 53000 - 32000 \leqslant LQ \leqslant 41000$)	**B1 FT**
		3

Question	Answer	Marks
6(c)	Sum of given 11 numbers is 433 000	**M1**
	Sum of 12 numbers, including new = $38\,500 \times 12 = 462\,000$	**M1**
	Difference = new salary = [$]29 000	**A1**
		3

Question	Answer	Marks
7(a)	$\frac{9!}{2!2!} = 90\,720$	**B1**
		1
7(b)	$\frac{6!}{2!}$	**M1**
	360	**A1**
		2

Question	Answer	Marks
7(c)	2 Es together $= \frac{8!}{2!}(= 20160)$	**M1**
	Es not together $= 90720 - 20160 = 70560$	**M1**
	Probability $= \frac{70560}{90720}$	**M1**
	$\frac{7}{9}$ or 0.778	**A1**
	Alternative method for question 7(c)	
	_ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ $\frac{7!}{2!} \times \frac{8 \times 7}{2} = 70560$	
	$7! \times k$ in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	**M1**
	Multiplying by ${}^{8}C_{2}$ OE	**M1**
	Probability $= \frac{70560}{90720}$	**M1**
	$\frac{7}{9}$ or 0.778	**A1**
		4

Question	Answer	Marks
7(d)	Scenarios are: E L _ _ _ $^5C_3$ 10 E E L _ _ $^5C_2$ 10 E _ _ _ _ $^5C_4$ 5 E E _ _ _ $^5C_3$ 10	**M1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total = 35	**A1**
		3

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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[Turn over

PUBLISHED

Generic Marking Principles

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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
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PUBLISHED

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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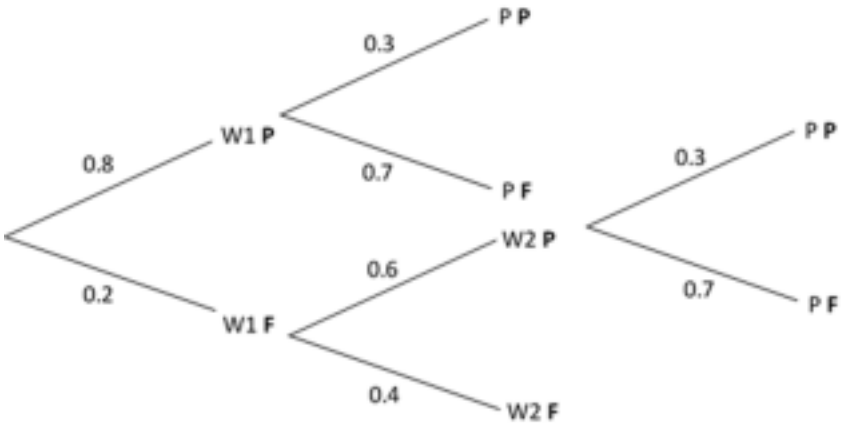
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

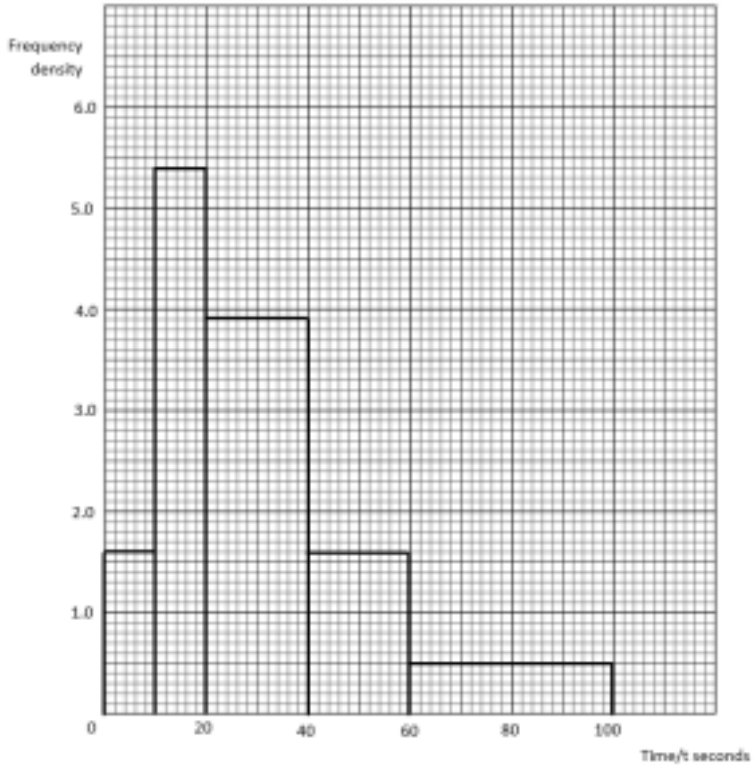
Question	Answer	Marks	Guidance
1	RRRRB ${}^8C_4 \times {}^4C_1 = 280$ BBBBR ${}^8C_1 \times {}^4C_4 = 8$ RRRRR ${}^8C_5 = 56$	M1	${}^8C_x \times {}^4C_y$ with $x + y = 5$. x, y both integers, $1 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant 4$ condone ${}^8C_1 \times 1$
		A1	Two correct outcomes evaluated
		M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
	[Total =] 344	A1	WWW, only dependent on 2nd M mark
		4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

Question	Answer	Marks	Guidance
2	$\left[P\left(\left(\frac{25.2-(25.5+0.50)}{0.4}\right)<z<\left(\frac{25.2-(25.2-0.50)}{0.4}\right)\right)\right]$ $= P\left(-\frac{0.5}{0.4}<z<\frac{0.5}{0.4}\right)$	M1	Use of $\pm$ Standardisation formula once; no continuity correction, σ^2, $\sqrt{\sigma}$
	$\left[=2\Phi(1.25)-1\right]$ $=2\times0.8944-1$	A1	For AWRT 0.8944 SOI
		M1	Appropriate area $2\Phi - 1$ OE, from final process, must be probability
	0.7888	A1	Accept AWRT 0.789
	Number of rods = 0.7888×500 = 394 or 395	B1FT	Correct or FT *their* 4SF (or better) probability, final answer must be positive integer, not 394.0 or 395.0, no approximation/rounding stated, only 1 answer
		5	

Question	Answer	Marks	Guidance
3(a)	$\left[\frac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
3(b)	_ _ _ L E D _ _ : With LED together: $\frac{6!}{2!}$	M1	$\frac{6!}{k}$ or $\frac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\frac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
3(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\frac{6!}{3!}$ [= 120]	*M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 6, 7 or 8, m an integer > 0
	[Probability =] $\frac{\textit{their}\,5040}{\textit{their}\,6720}$	DM1	Denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\frac{7! \times 2}{3!}$ [= 5040]	*M1	*Their* 6720 – k, k a positive integer
		*M1	$(m-)\frac{7! \times k}{3!}, k = 1, 2$

Question	Answer	Marks	Guidance
	[Probability =] $\frac{their\,5040}{their\,6720}$	DM1	With denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using '1 – Probability of arrangements with A and D together': $\frac{7!\times 2}{3!}$ [= 1680]	*M1	$\frac{7!\times k}{3!}, k = 1, 2$
	[Probability =] $\frac{their\,1680}{their\,6720}$	*M1	With denominator = *their* **(a)** or correct
	$1 - \frac{their\,1680}{their\,6720}$	DM1	$1 - m$, $0 < m < 1$, dependent on at least one M mark already gained
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
		4	

Question	Answer	Marks	Guidance
4(a)		B1	Fully correct labelled tree diagram for each pair of branches clearly identifying written and practical, pass and fail for each intersection (no additional branches)
		B1	'One written test' branch all probabilities (or %) correct
		B1	'Two written tests' branch all probabilities (or %) correct, condone additional branches after W2F with probabilities 1 for PF and 0 for PP
		3	
4(b)	[P(W1P) × P(PP) + P(W1F) × P(W2P) × P(PP)] $0.8\times0.3+0.2\times0.6\times0.3$	M1	Consistent with *their* tree diagram or correct
	0.276 or $\frac{69}{250}$	A1	
		2	
4(c)	$P(W1\mid P)=\frac{P(W1\cap\text{Practical})}{P(\text{getting place})}=\frac{0.8\times0.3}{their(b)}\left[=\frac{0.24}{0.276}\right]$	M1	Correct expression or FT *their* **(b)**
	$\frac{20}{23}$ or 0.87[0]	A1	
		2	

Question	Answer	Marks	Guidance
5(a)	<table><tr><td>Class width</td><td>10</td><td>10</td><td>20</td><td>20</td><td>40</td></tr><tr><td>Frequency Density</td><td>1.6</td><td>5.4</td><td>3.9</td><td>1.6</td><td>0.5</td></tr></table> 	M1	At least 4 frequency densities calculated, accept unsimplified. May be read from graph using *their* scale, 3SF or correct
		A1	All heights correct on graph
		B1	Bar ends at 0, 10, 20 …, etc. with a horizontal linear scale with at least 3 values indicated, 0 ⩽ horizontal axis ⩽ 100
		B1	Axes labelled: Frequency density (fd), time (t) and seconds. Linear vertical scale, with at least 3 values indicated 0 ⩽ vertical axis ⩽ 5.4
		4	

Question	Answer	Marks	Guidance
5(b)	$\text{Mean} = \left[\dfrac{16\times5+54\times15+78\times30+32\times50+20\times80}{200}\right]$ $= \dfrac{80+810+2340+1600+1600}{200}$	M1	Uses at least 4 midpoint attempts (e.g. 5 ± 0.5). Accept unsimplified expression, denominator either correct or *their* Σfrequencies
	$\left[\dfrac{6430}{200} =\right] 32\dfrac{3}{20}$ or 32.15	A1	Accept 32.2
		2	
5(c)	A value in correct UQ (40–60) – a value in correct LQ (10–20)	M1	
	Greatest possible value is 60 – 10 = 50	A1	Condone $49.\dot{9}$
		2	

Question	Answer	Marks	Guidance
6(a)	$1 - P(10, 11, 12) = 1 - ({}^{12}C_{10}\,0.6^{10}\,0.4^{2} + {}^{12}C_{11}\,0.6^{11}\,0.4^{1} + {}^{12}C_{12}\,0.6^{12}\,0.4^{0})$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	$P(0,1,2,3,4,5,6,7,8,9) = {}^{12}C_0 0.6^0\,0.4^{12} + {}^{12}C_1\,0.6^1\,0.4^{11} + \ldots\ldots\ldots\ldots {}^{12}C_9\,0.6^9\,0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	

Question	Answer	Marks	Guidance
6(b)	[Mean =] 0.6×150 [= 90]; [Variance =] $0.6 \times 150 \times 0.4$ [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X < 81) = P\left(Z < \frac{80.5 - 90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833) = 1 - 0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
6(c)	$np = 90$, $nq = 60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

Question	Answer	Marks	Guidance
7(a)	$P(X = 3) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$	M1	$\frac{m}{7} \times \frac{n}{6} \times \frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	

Question	Answer	Marks	Guidance
7(b)	$\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 1 & 2 & 3 & 4 & 5 \\ \hline p & \frac{15}{35} & \frac{10}{35} & \frac{6}{35} & \frac{3}{35} & \frac{1}{35} \\ \hline \end{array}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X = 3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X = 3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
7(c)	$[E(X) = 1\times\frac{15}{35} + 2\times\frac{10}{35} + 3\times\frac{6}{35} + 4\times\frac{3}{35} + 5\times\frac{1}{35}]$ $E(X) = \frac{15+20+18+12+5}{35}\left[= \frac{70}{35} = 2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.
	$\text{Var}(X) = \left[\frac{1^2\times15 + 2^2\times10 + 3^2\times6 + 4^2\times3 + 5^2\times1}{35} - 2^2 = \right]$ $\frac{15+40+54+48+25}{35} - 2^2$	M1	Appropriate variance formula using *their* $(\text{E}(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[= \frac{182}{35} - 4\right] = \frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	6	B1	WWW
		1	
1(b)	$\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+\left(\frac{5}{6}\right)^6\frac{1}{6}$	M1	$p^3(1-p)+p^4(1-p)+p^5(1-p)+p^6(1-p)$, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
	Alternative method for Question 1(b)		
	$\left(\frac{5}{6}\right)^3-\left(\frac{5}{6}\right)^7$	M1	p^3-p^7, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
		2	
1(c)	$1-\left(\frac{5}{6}\right)^9$	M1	$1-p^n$, $0<p<1$, $n=9, 10$
	0.806	A1	
	Alternative method for Question 1(c)		
	$\frac{1}{6}+\frac{1}{6}\left(\frac{5}{6}\right)+\frac{1}{6}\left(\frac{5}{6}\right)^2+\cdots+\frac{1}{6}\left(\frac{5}{6}\right)^8$	M1	$p+p(1-p)+p(1-p)^2+p(1-p)^3+p(1-p)^4+p(1-p)^5+p(1-p)^6+p(1-p)^7+p(1-p)^8\,(+p(1-p)^9)$, $0<p<1$ As per answer for minimum terms shown
	0.806	A1	
		2	

Question	Answer	Marks	Guidance
2	$\left[P(X>1.1)=\frac{72}{2000}(=0.036)\right]$ $z=\pm 1.798$	B1	$1.79 < z \leqslant 1.80$, $-1.80 \leqslant z < -1.79$ seen
	$\frac{1.1-1.04}{\sigma}=1.798$ $\left[\frac{0.06}{\sigma}=1.798\right]$	B1	1.1 and 1.04 substituted in ±standardisation formula, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Equate *their* ±standardisation formula to a z-value and to solve for the appropriate area leading to final answer (expect $\sigma < 0.5$). $\left(\text{Accept} \pm \frac{0.06}{\sigma} = z - \text{value}\right)$
	$\sigma = 0.0334$	A1	$0.03335 \leq \sigma \leq 0.0334$. At least 3 3s.f.
		4	

Question	Answer	Marks	Guidance
3(a)	P(not late) = 0.4 × 0.45 + 0.35 × 0.3 + 0.25 × (1 – x) or P(late) = 0.4 × 0.55 + 0.35 × 0.7 + 0.25x	M1	$0.4\times p+0.35\times q+0.25\times r$, p = 0.45, 0.55, q = 0.3, 0.7 and r = (1 – x), x
	0.18 + 0.105 + 0.25 (1 – x) = 0.48 or 0.22 + 0.245 + 0.25x = 0.52	A1	Linear equation formed using sum of 3 probabilities and 0.48 or 0.52 as appropriate. Accept unsimplified.
	x = 0.22	A1	Final answer
		3	
3(b)	$\left[P\left(train\middle\vert late\right)=\frac{P\left(train\cap late\right)}{P\left(late\right)}\right]$ $=\frac{0.35\times0.7}{1-0.48}$ or $\frac{0.35\times0.7}{0.4\times0.55+0.35\times0.7+0.25\times their\,0.22}$	B1	0.35×0.7 or 0.245 seen as numerator of fraction
		M1	P(late) seen as a denominator with *their* probability as numerator (Accept $\frac{their\ p}{0.52}$ or $\frac{their\ p}{0.22+0.245+0.25\times their\,0.22}$)
	= 0.471 or $\frac{49}{104}$	A1	
		3	

Question	Answer	Marks	Guidance
4(a)	X: –1, 0, 1, 2, 3 $P(X)$: $\frac{1}{9}$, $\frac{2}{9}$, $\frac{1}{9}$, $\frac{3}{9}$, $\frac{2}{9}$	B1	Table with correct X values and at least one probability Condone any additional X values if probability stated as 0.
		B1	2 correct probabilities linked with correct outcomes, may not be in table.
		B1	3 further correct probabilities linked with correct outcomes, may not be in table. **SC** if less than 2 correct probabilities seen, award **SCB1** for **s**um of *their* 4 or 5 probabilities in table = 1
		3	
4(b)	$\left[E(X)=\frac{-1\times1+(0\times2)+1\times1+2\times3+3\times2}{9}=\right]$ $\frac{-1+1+6+6}{9}$	M1	May be implied by use in variance, accept unsimplified expression. FT *their* table if *their* 3 or more probabilities sum to 1 or 0.999
	[Var (X) =] $\left[\frac{-1^2\times1+(0^2\times2)+1^2\times1+2^2\times3+3^2\times2}{9}-(\textit{their}\,E(X))^2\right]$ $\frac{1+0+1+12+18}{9}-(\textit{their}\,E(X))^2$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table even if *their* 3 or more probabilities not summing to 1.
	$E(X)=\frac{4}{3}$ or 1.33 and $Var(X)=\frac{16}{9}$ or 1.78	A1	Answers for E(X) and Var(X) must be identified
		3	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1

Question	Answer	Marks	Guidance
5(a)	$[(0.7)^3 =]0.343$	B1	Evaluated WWW
	Alternative method for Question 5(a)		
	$[(0.15)^3 + {}^3C_1(0.15)^2(0.55) + {}^3C_2(0.15)(0.55)^2 + (0.55)^3 =] 0.343$	B1	Evaluated WWW
		1	
5(b)	$1 - (0.85^9 + {}^9C_1\, 0.15^1\, 0.85^8 + {}^9C_2\, 0.15^2\, 0.85^7)$ $[1 - (0.231617 + 0.367862 + 0.259667)]$	M1	One term: ${}^9C_x\, p^x\, (1 - p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
	Alternative method for Question 5(b)		
	${}^9C_3\, 0.15^3\, 0.85^6 + {}^9C_4\, 0.15^4\, 0.85^5 + {}^9C_5\, 0.15^5\, 0.85^4 + {}^9C_6\, 0.15^6\, 0.85^3 +$ ${}^9C_7\, 0.15^7\, 0.85^2 + {}^9C_8\, 0.15^8\, 0.85 + 0.15^9$	M1	One term: ${}^9C_x\, p^x\, (1 - p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
		3	

Question	Answer	Marks	Guidance
5(c)	Mean $=[60\times0.15=]9$ Variance $=[60\times0.15\times0.85=]7.65$	B1	Correct mean and variance, allow unsimplified. ($2.765 \le \sigma \le 2.77$ imply correct variance)
	$\left[(X\ge12)=\right]P\left(Z>\dfrac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1-\Phi(0.9039)=1-0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

Question	Answer	Marks	Guidance
6(a)	$\dfrac{8!}{2!3!}$	M1	$\dfrac{8!}{k!\times m!}$ $k = 1$ or 2, $m = 1$ or 3, not $k = m = 1$ no additional terms
	3360	A1	
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1** Arrangements Rs at ends – Arrangements Rs at ends and Os together		
	[Os not together $=]\ \frac{6!}{3!} - 4!$	M1	$\frac{6!}{k!} - m$, $1 \leqslant k \leqslant 3$, m an integer, condone $2\times\left(\frac{6!}{k!}\right) - m$.
		M1	$w - 4!$ or $w - 24$, w an integer Condone $w - 2 \times 4!$
	96	A1	
	Method 2 identified scenarios R _ _ _ R, Arrangement No Os together + 2Os and a single O		
	${}^4C_3 \times 3! + {}^4C_2 \times 2 \times 3!$	M1	${}^4C_3 \times 3! + r$ or $4\times 3! + r$ or ${}^4P_3 \times 3! + r$, r an integer. Condone $2 \times {}^4C_3 \times 3! + r$. $2 \times 4\times 3! + r$ or $2 \times {}^4P_3 \times 3! + r$.
		M1	$q + {}^4C_2 \times 3! \times k$ or $q + {}^4P_2 \times 3! \times k$, $k = 1,2$, q an integer
	[24 + 72 =] 96	A1	
		3	
6(c)	**Method 1** Identified scenarios		
	OORR ${}^3C_2 \times {}^2C_2 \times \left[{}^3C_0\right] = 3\times 1 = 3$ ORR_ ${}^3C_1 \times {}^2C_2 \times {}^3C_1 = 3\times 1\times 3 = 9$ OOR_ ${}^3C_2 \times {}^2C_1 \times {}^3C_1 = 3\times 2\times 3 = 18$ OR_ _ ${}^3C_1 \times {}^2C_1 \times {}^3C_2 = 3\times 2\times 3 = 18$ OOOR ${}^3C_3 \times {}^2C_1 \times \left[{}^3C_0\right] = 1\times 2 = 2$	B1	Outcomes for 2 identifiable scenarios correct, accept unsimplified.
		M1	Add 4 or 5 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability $= \frac{50}{{}^8C_4}$	M1	$\frac{\textit{their}'50'}{{}^8C_4}$, accept numerator unevaluated

Question	Answer	Marks	Guidance
6(c) cont'd	$\frac{50}{70}$ or 0.714	A1	
	Method 2 Identified outcomes		
	ORTM $^3C_1 \times {}^2C_1 = 6$ ORTW $^3C_1 \times {}^2C_1 = 6$	B1	Outcomes for 5 identifiable scenarios correct, accept unsimplified.
	ORMW $^3C_1 \times {}^2C_1 = 6$ ORRM $^3C_1 \times {}^2C_2 = 3$ ORRW $^3C_1 \times {}^2C_2 = 3$ ORRT $^3C_1 \times {}^2C_2 = 3$ OROR $^3C_2 \times {}^2C_2 = 3$ OROT $^3C_2 \times {}^2C_1 = 6$ OROM $^3C_2 \times {}^2C_1 = 6$ OROW $^3C_2 \times {}^2C_1 = 6$ OROO $^3C_3 \times {}^2C_1 = 2$	M1	Add 9, 10 or 11 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{{}^8C_4}$	M1	$\frac{\textit{their}'50'}{{}^8C_4}$, accept numerator unevaluated.
	$\frac{50}{70}$ or 0.714	A1	
		5	

Question	Answer	Marks	Guidance
7(a)	Includes all data	B1	Reference to *either* including all/raw data or further statistical processes are possible that cannot be found using data from box-and-whisker, eg frequency, mean, mode or standard deviation **not** only median, IQR, range or spread which can be found from both.
		1	
7(b)	<table><tr><td colspan="3">Amazons</td><td></td><td colspan="5">Giants</td></tr><tr><td></td><td></td><td>8</td><td>17</td><td>5</td><td></td><td></td><td></td><td></td></tr><tr><td>4</td><td>2</td><td>1</td><td>18</td><td>2</td><td>4</td><td>7</td><td>9</td><td></td></tr><tr><td>8</td><td>6</td><td>0</td><td>19</td><td>2</td><td>3</td><td>5</td><td>5</td><td>5</td></tr><tr><td>5</td><td>2</td><td>1</td><td>20</td><td>4</td><td></td><td></td><td></td><td></td></tr><tr><td></td><td></td><td>5</td><td>21</td><td></td><td></td><td></td><td></td><td></td></tr></table> Key: 1|18|2 means 181 cm for Amazons and 182 cm for Giants	B1	Correct stem can be upside down, ignore extra values
		B1	Correct Amazons labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.
		B1	Correct Giants labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation.
		B1	Correct single key for their diagram, need both teams identified and 'cm' stated at least once here or in leaf headings or title. **SC** for if 2 separate diagrams drawn, award **SCB1** if both keys meet these criteria (Max B1, B0, B0, B1)
		4	
7(c)	[UQ = 202 (cm), LQ = 182 (cm)] [IQR =] 202 – 182 = 20 (cm)	M1	$201 \leqslant UQ \leqslant 205 - 181 \leqslant LQ \leqslant 184$
		A1	WWW
		2	

Question	Answer	Marks	Guidance
7(d)	$[\Sigma_{11} = 2132$ $\Sigma_{15} = 191.2 \times 15 = 2868\ ]$	B1	Both Σ_{11} and Σ_{15} found. Accept unevaluated.
	their 2868 = *their* 2132 + (180 + 185 + 190) + h	M1	Forming an equation for the height using *their* Σ_{11} and Σ_{15}.
	181 (cm)	A1	
	Alternative method for Question 7(d)		
	$[\Sigma_{15} = 191.2 \times 15 = 2868$ $\Sigma_{15} = 2687 + h\ ]$	B1	Σ_{15} found using the mean and raw data methods. Accept unevaluated.
	their 2868 = *their* 2687 + h	M1	Forming an equation for the height using *their* Σ_{15} expressions.
	181 (cm)	A1	
	Alternative method for Question 7(d)		
	$[\Sigma_{15} = 2687 + h$ $\frac{\Sigma_{15}}{15} = 191.2\ ]$	B1	Σ_{15} found using raw data method and statement on calculating new mean. Accept unevaluated.
	$\frac{\textit{their}\ 2687 + h}{15} = 191.2$	M1	Forming an equation for the height using *their* Σ_{15} expressions
	181 (cm)	A1	
		3	**N.B.** All methods can be presented as a logical numerical argument which can be condoned if clear.

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	60	**B1**	Accept 60 or 61. No decimals
		1	
1(b)	65% of 160 = 104	**M1**	0.65 × 160 (=104) seen unsimplified or implied by use on graph
	136 (cm)	**A1**	Use of graph must be seen. **SCB1** correct value (136 only) if neither 104 nor use of graph are evident
		2	
1(c)	UQ: 150 LQ: 76 IQR = 150 – 76 = 74 [cm]	**M1**	UQ – LQ ; 148 ⩽ UQ ⩽ 152; 74 ⩽ LQ ⩽ 78.
		A1	Must be from 150 - 76
		2	

Question	Answer	Marks	Guidance
2	$p + p + 0.1 + q + q = 1$	**B1**	Sum of probabilities = 1
	$0.1 + 2q = 3(2p)$	**B1**	Use given information
	Attempt to solve two correct equations in p and q	**M1**	**Either** use of Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	**A1**	CAO, both WWW
		4	

Question	Answer	Marks	Guidance
3(a)	Mean height $= \frac{\Sigma x + \Sigma y}{6+11} = \frac{1050+1991}{6+11} = \frac{3041}{17}$	**M1**	Use of appropriate formula with values substituted, accept unsimplified.
	178.9	**A1**	Allow 178.88, $178\frac{15}{17}$, 179
		2	

Question	Answer	Marks	Guidance
3(b)	$\frac{\Sigma x^2+\Sigma y^2}{6+11}=\frac{193700+366400}{6+11}$	**M1**	Use of appropriate formula with values substituted, accept unsimplified.
	$\text{Sd}^2=\frac{560100}{17}-\textit{their}\,178.88^2\,[=948.289]$	**M1**	Appropriate variance formula using *their* mean^2, accept unsimplified expression.
	Standard deviation = 30.8	**A1**	Accept 30.7
		3	

Question	Answer	Marks	Guidance
4(a)	[Possible cases: 1 1 2, 1 2 1, 2 1 1] Probability = $\left(\frac{1}{6}\right)^3\times 3$	**M1**	$\left(\frac{1}{6}\right)^3\times k$, where k is an integer.
		M1	Multiply a probability by 3, not +, – or ÷
	$\frac{1}{72}$	**A1**	Accept $\frac{3}{216}$ or $0.013\dot{8}$ or 0.0139
		3	
4(b)	$\text{P}(18)=\left(\frac{1}{6}\right)^3\left[=\frac{1}{216}\right]$	**B1**	
	$\text{P}(18 \text{ on 5th throw})=\left(\frac{215}{216}\right)^4\times\frac{1}{216}$	**M1**	$(1-p)^4p$, $0<\textit{their}\ p<1$
	0.00454	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	$z_1 = \frac{4-\mu}{\sigma} = -1.378$	**B1**	$1.378 \leqslant z_1 \leq 1.379$ or $-1.379 \leqslant z_1 \leqslant -1.378$
	$z_2 = \frac{10-\mu}{\sigma} = 0.842$	**B1**	$0.841 \leqslant z_2 \leqslant 0.842$ or $-0.842 \leqslant z_2 \leqslant -0.841$
	Solve to find at least one unknown: $\frac{4-\mu}{\sigma} = -1.378$ $\frac{10-\mu}{\sigma} = 0.842$	**M1**	Use of ±standardisation formula once with μ, σ, a z-value and 4 or 10, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve two equations in μ and σ.
	$\sigma = 2.70 \quad \mu = 7.72$	**A1**	$2.70 \leqslant \sigma \leqslant 2.71 \quad 7.72 \leqslant \mu \leqslant 7.73$
		5	
5(b)	$\Phi(2) - \Phi(-2) = 2\Phi(2) - 1$	**M1**	Identifying 2 and –2 as the appropriate z-values
	$2 \times \textit{their}\, 0.9772 - 1$	**B1**	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.9544 or 0.9545	**A1**	Accept AWRT 0.954
	$0.9544 \times 800 = 763.52$ 763 or 764	**B1 FT**	FT *their* 4SF (or better) probability, final answer must be positive integer
		4	

Question	Answer	Marks	Guidance
6(a)	$\frac{11!}{2!3!}$	**M1**	11! alone on numerator – must be a fraction. $k! \times m!$ on denominator, $k = 1, 2$, $m = 1, 3, 1$ can be implied but cannot both = 1. No additional terms
	3326400	**A1**	Exact value only
		2	
6(b)	8! = 40320	**B1**	Evaluate, exact value only
		1	
6(c)	$\frac{9!}{3!} \times 7$	**M1**	$\frac{9!}{3!} \times k$ seen, k an integer > 0, no +, – or ÷
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	**A1**	Exact value only
	Alternative method for Question 6(c)		
	${}^9C_3 \times 7! \left(\times \frac{3!}{3!}\right)$	**M1**	$9C3 \times k$ seen, k an integer > 0, no + or –
		M1	$7! \times k$ seen, , k an integer > 0, no + or –
	423360	**A1**	Exact value only but there must be evidence of $\times \frac{3!}{3!}$

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	$3\times 7\times\frac{8!}{2!}$	**M1**	$3\times\frac{8!}{2!}\times k$ seen, k an integer > 0, no + or –
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	**A1**	Exact value only
	Alternative method for Question 6(c)		
	$7\times\frac{2}{11}\times\frac{9}{10}\times\frac{8}{9}\times\frac{7}{8}\times\frac{1}{7}\times$ total no. of arrangements	**M1**	Product of correct five fractions × k seen, k an integer > 0, no + or –
		M1	7×'total no of arrangements' ×k seen, k an integer > 0, no + or –
	423360	**A1**	Exact value only
	Alternative method for Question 6(c)		
	No E between the Rs $-\frac{{}^6C_3\times 3!\times 7!}{3!}=100800$ 1E between the Rs $-\frac{{}^6C_2\times 3!\times 7!}{2!}=226800$ 2Es between the Rs $-{}^6C_1\times 3!\times 7!=90720$ 3Es between the Rs $-7!=5040$	**M1** **M1**	Finding the correct number of ways for no, 1 or 2 Es between the Rs, accept unsimplified. Adding the number of ways for 3 or 4 correct scenarios
	$\left[\text{Total}=7!\times(20+45+18+1)=7!\times 84=\right]423360$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
6(d)	E E R _ _ $^6C_2 = 15$ E E R R _ $^6C_1 = 6$ E E E R _ $^6C_1 = 6$ E E E R R $^6C_0 = 1$	**M1**	Identifying four correct scenarios only.
		B1	Correct number of selections unsimplified for 2 or more scenario.
		M1	Adding the number of selections for 3 or 4 identified correct scenarios only, accept unsimplified. $^3C_x \times {}^2C_y \times {}^6C_z$, $x+y+z=5$ correctly identifies x Es and y Rs
	[Total =] 28	**A1**	WWW, only dependent upon 2nd M mark.
	Alternative method for Question 6(d) – Fixing EER first. No other scenarios can be present anywhere in solution.		
	E E R ^ ^ = $^8C_2$	**M1**	^8C_x seen alone or $^8C_x \times k$, , $k = 1$ or 2, $0<x<8$ Condone ^8P_x or $^8P_x \times k$, $k = 1$ or 2, $0<x<8$
		B1	$^8C_2 \times k$, $k = 1$ or 2 OE
		M1	$^8C_2 \times k$, $k = 1$ OE and no other terms
	[Total =] 28	**A1**	Value stated
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\frac{40}{800}$ or $\frac{1}{20}$ or 0.05	**B1**	
		1	
7(a)(ii)	$\frac{177}{223+177+40}$	**M1**	*Their* 223 + 177 + 40 seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	**A1**	CAO
	Alternative method for Question 7(a)(ii)		
	$P(\text{G} \mid \text{S}) = \frac{P(G \cap S)}{P(S)} = \frac{\frac{177}{800}}{\frac{223+177+40}{800}} = \frac{\frac{177}{800}}{\frac{440}{800}} = \frac{\frac{177}{800}}{\frac{11}{20}}$ or 0.55	**M1**	*Their* P(S) seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	**A1**	CAO
		2	
7(b)(i)	P(0, 1, 2) = $^{10}\text{C}_0 (0.35)^0 (0.65)^{10} + {}^{10}\text{C}_1 (0.35)^1 (0.65)^9 + {}^{10}\text{C}_2 (0.35)^2 (0.65)^8$	**M1**	One term: $^{10}\text{C}_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any $0<p<1$
	0.013463 + 0.072492 + 0.17565	**A1**	Correct unsimplified expression, or better
	0.262	**A1**	
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	Mean = $120\times0.35\left[=42\right]$ Variance = $120\times0.35\times0.65\left[=27.3\right]$	**B1**	Correct mean and variance seen, allow unsimplified
	P(X >32) = P($Z >$ $\frac{32.5-42}{\sqrt{27.3}}$) = P($Z$ > - 1.818)	**M1**	Substituting *their* mean and variance into ±standardisation formula (any number), condone σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 31.5 or 32.5
	$\Phi\left(1.818\right)$	**M1**	Appropriate area Φ, from final process, must be probability
	0.966	**A1**	$0.965 \leqslant p \leqslant 0.966$
		5	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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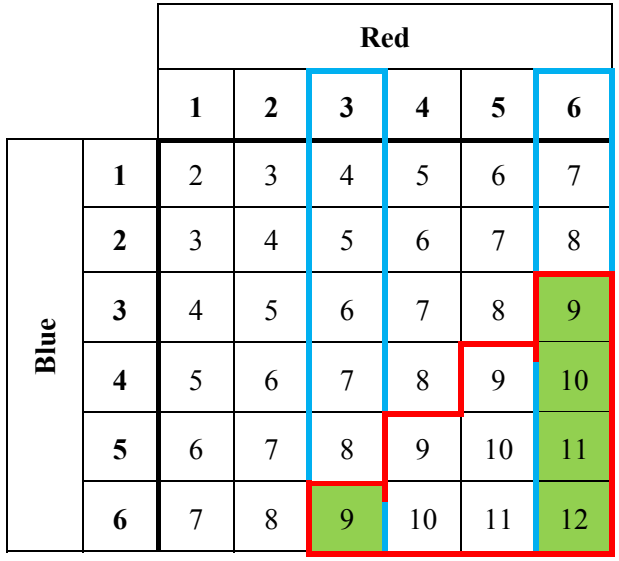
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)		**M1**	Complete outcome space or or listing A and B outcomes or listing A∩B outcomes
	$P(A \cap B) = \frac{5}{36}$	**A1**	With evidence
		2	

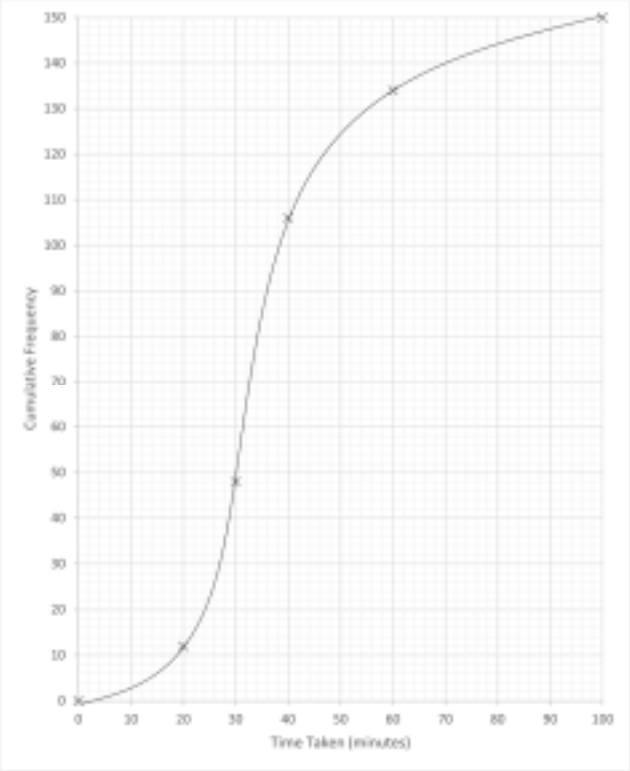
Question	Answer	Marks	Guidance
1(b)	$P(A) \times P(B) = \frac{1}{3} \times \frac{10}{36}$	**M1**	*Their* $\frac{1}{3} \times$ *their* $\frac{10}{36}$ seen
	$\frac{5}{54} \neq \frac{5}{36}$ so not independent	**A1**	$\frac{5}{54}, \frac{5}{36}$, $P(A) \times P(B)$ and $P(A \cap B)$ seen in workings and correct conclusion stated Condone $\frac{5}{36}$ being stated in **(a)**
	Alternative method for question 1(b)		
	$P(B\|A) = P(B)$ $P(B\|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{36}}{\frac{1}{3}}$	**M1**	OE, $\frac{\textit{their}\,1(a)}{\textit{their}\,P(A)}$ seen
	$\frac{5}{12} \neq \frac{5}{18}$ so not independent	**A1**	$P(A\|B)$, $P(B)$, $\frac{5}{12}, \frac{5}{18}$ seen in workings and correct conclusion stated Condone $\frac{5}{18} \equiv \frac{10}{36}$ being identified in **(a)**
		2	

Question	Answer	Marks	Guidance
2(a)	$0{\cdot}6 \times 0{\cdot}7 + 0{\cdot}4(1 - x) = 0{\cdot}58$ $\equiv 0{\cdot}42 + 0{\cdot}4(1 - x) = 0{\cdot}58$	**M1**	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}58$; $a = 0{\cdot}3, 0{\cdot}7$, b = x, $(1 - x)$
		B1	Single correct product seen, condone 0·42, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
	Alternative method for question 2(a)		
	$0{\cdot}6 \times 0{\cdot}3 + 0{\cdot}4x = 0{\cdot}42$ $\equiv 0{\cdot}18 + 0{\cdot}4x = 0{\cdot}42$	**M1**	Equation of form 0·6 x a + 0·4 x b = 0·42; $a = 0{\cdot}3, 0{\cdot}7$, b = x, $(1 - x)$
		B1	Single correct product seen, condone 0·18, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
		3	
2(b)	$(0.6 \times 0.3)^2$	**M1**	$(a \times b)^2$, $a = 0{\cdot}6, 0{\cdot}4$ and $b = 0{\cdot}7, 0{\cdot}3, x, (1–x)$ or $0{\cdot}18^2$, alone.
	0.0324	**A1**	
		2	
3(a)	$P(X > 6) = 0.75^6$	**M1**	p^n, $n = 6, 7$ $0 < p < 1$
	$0.178, \frac{729}{4096}$	**A1**	0·17797…
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">3(b)</td><td>$1 - P(0, 1, 2) = 1 - (0.75^{10} + {}^{10}C_1\ 0.25^1\ 0.75^9 + {}^{10}C_2\ 0.25^2\ 0.75^8)$</td><td>M1</td><td>Binomial term of form ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0, 10$</td></tr>
<tr><td>$1 - (0{\cdot}0563135 + 0{\cdot}1877117 + 0{\cdot}2815676)$</td><td>A1</td><td>Correct unsimplified expression</td></tr>
<tr><td>$0{\cdot}474$</td><td>A1</td><td>$0{\cdot}474 \leqslant p \leqslant 0{\cdot}4744$</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="5">4(a)</td><td rowspan="5"><table><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>prob</td><td>$\frac{7}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{3}{16}$</td><td>$\frac{1}{16}$</td></tr></table></td><td>B1</td><td><table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2</td><td>3</td></tr><tr><td>2</td><td>1</td><td>2</td><td>1</td><td>2</td></tr><tr><td>3</td><td>2</td><td>1</td><td>3</td><td>1</td></tr><tr><td>4</td><td>3</td><td>2</td><td>1</td><td>4</td></tr></table>Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated’</td></tr>
<tr><td>B1</td><td>One probability (linked with correct score) correct</td></tr>
<tr><td>B1</td><td>2 more probs (linked with correct scores) correct</td></tr>
<tr><td>B1 FT</td><td>4^{th} prob correct, FT sum of 3 or 4 terms = 1</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	$\text{P}(2\|\text{even}) = \dfrac{\frac{5}{16}}{\frac{6}{16}}$	**M1**	$\dfrac{\textit{their}\,\text{P}(2)}{\textit{their}\,\text{P}(2)+\textit{their}\,\text{P}(4)}$ seen or correct outcome space.
	$\frac{5}{6}$ or 0·833	**A1**	
		2	
5(a)	$\text{P}(X > 4.2) = \text{P}(z > \dfrac{4.2-3.5}{0.9})$ $= \text{P}(z > 0.7778)$	**M1**	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 – 0·7818	**M1**	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	0·218	**A1**	
		3	
5(b)	$z = -1.282$	**B1**	±1.282 seen (critical value)
	$\dfrac{t-3.5}{0.9} = -1.282$	**M1**	An equation using ±standardisation formula with a z-value, condone $\sqrt{\sigma}$, σ^2 and continuity correction
	$t = 2.35$	**A1**	AWRT, only dependent on M mark
		3	

Question	Answer	Marks	Guidance
5(c)	P(2.8 < X < 4.2) = 1 – 2 × *their* **5(a)** ≡ 2(1 – *their* **5(a)**) – 1 ≡ 2(0·5 – *their* **5(a)**) = 0·5636	**B1 FT**	FT from *their* **5(a)** < 0.5 or correct Accept unevaluated probability OE Accept 0·564
	Number of days = 365 × 0·5636 = 205·7	**M1**	365 × *their p*
	So, 205 (days)	**A1 FT**	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
	Alternative method for question 5(c)		
	$P\left(\frac{2.8-3.5}{0.9} < z < \frac{4.2-3.5}{0.9}\right)$ $= \Phi(0.7778) - (1 - \Phi 0.7778)$ = 0·7818 – (1 – 0·7818) = 0·5636	**B1**	0·5635 < $p \leqslant$ 0·564 OE
	Number of days = 365 × 0·5636 = 205·7	**M1**	365 × *their p*
	So, 205 (days)	**A1 FT**	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
		3	

Question	Answer	Marks	Guidance
6(a)		**M1**	At least 4 points plotted at upper end points, with both scales linear with at least 3 values indicated
	Correct cumulative frequency curve	**A1**	All plotted correctly with curve drawn joined to (0, 0), axes labelled cumulative frequency, time, minutes
		2	
6(b)	$150 \times 0{\cdot}76 = 114$	**M1**	114 SOI, may be on graph
	$k = 45$ (mins)	**A1 FT**	Clear indication that *their* graph has been used, tolerance ±1mm
		2	

Question	Answer	Marks	Guidance
6(c)	Frequencies: 12 36 58 28 16	**B1**	Correct frequencies seen
	Mean = $\frac{10\times12+25\times36+35\times58+50\times28+80\times16}{150}$	**B1**	At least 4 correct midpoints seen and used
	$\frac{120+900+2030+1400+1280}{150}$	**M1**	Correct formula with *their* midpoints (not upper boundary, lower boundary, class width or frequency density).
	38.2, $38\frac{1}{5}$	**A1**	
	Variance = $\frac{12\times10^2+36\times25^2+58\times35^2+28\times50^2+16\times80^2}{150}-mean^2$ $=\frac{1200+22500+71050+70000+102400}{150}-mean^2$	**M1**	Substitute *their* midpoints and frequencies (condone use of cumulative frequency) in correct variance formula, must have '– *their* mean^2'
	(Standard deviation = $\sqrt{321.76}$) = 17.9	**A1**	
		6	
7(a)	$\frac{8!}{2!}$	**M1**	$\frac{8!}{k}\equiv\frac{7!\times8}{k}$, where $k\in\mathbb{N}$, $\frac{a!}{2(!)}$, where $a\in\mathbb{N}$
	20160	**A1**	
		2	

Question	Answer	Marks	Guidance
7(b)	Total number of ways: $\frac{10!}{2!3!}\ (=302\,400)$ (A)	**B1**	Accept unsimplified
	With Ps together: $\frac{9!}{3!}$ (= 60 480) (B)	**B1**	Accept unsimplified
	With Ps not together: 302 400 – 60 480	**M1**	$\frac{10!}{m}-\frac{9!}{n}$, *m, n* integers or (A) – (B) if clearly identified
	241 920	**A1**	
	Alternative method for question 7(b)		
	$\frac{8!}{3!}$	**B1**	$k \times 8!$ in numerator, k a positive integer, no ±
		B1	$m \times 3!$ in denominator, m a positive integer, no ±
	$\times\frac{9\times8}{2}$	**M1**	*Their* $\frac{8!}{3!}$ multiplied by ${}^9C_2$ or ${}^9P_2$ no additional terms
	241 920	**A1**	Exact value, WWW
		4	

Question	Answer	Marks	Guidance
7(c)	Probability = $\dfrac{\text{Number of ways Es at beginning and end}}{\text{Total number of ways}}$ Probability = $\dfrac{\frac{8!}{2!}}{\frac{10!}{2!\times 3!}} = \dfrac{20160}{302400}$	**M1**	$\dfrac{\left(\frac{8!}{k!}\right)}{\frac{10!}{k!l!}}$ $1 \leqslant k, l \in \quad \mathbb{N} \leqslant 3$, FT denominator from **7(b)** or correct
	$\frac{1}{15}$, 0·0667	**A1**	
	Alternative method for question 7(c)		
	Probability = $\frac{3}{10}\times\frac{2}{9}$	**M1**	$\frac{a}{10}\times\frac{a-1}{9}a=3,2$
	$\frac{1}{15}$, 0·0667	**A1**	
	Alternative method for question 7(c)		
	Probability = $\frac{1}{10}\times\frac{1}{9}\times 3!$	**M1**	$\frac{1}{10}\times\frac{1}{9}\times m!, m=3,2$
	$\frac{1}{15}$, 0.0667	**A1**	
		2	

Question	Answer	Marks	Guidance
7(d)	Scenarios: P E E E $^5C_0 = 1$ P E E _ $^5C_1 = 5$ P E _ _ $^5C_2 = 10$ P _ _ _ $^5C_3 = 10$	**M1**	^5C_x seen alone, $1 \leqslant x \leqslant 4$
		M1	Summing the number of ways for 3 or 4 correct scenarios (can be unsimplified), no incorrect scenarios
	Total = 26	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

[Turn over

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

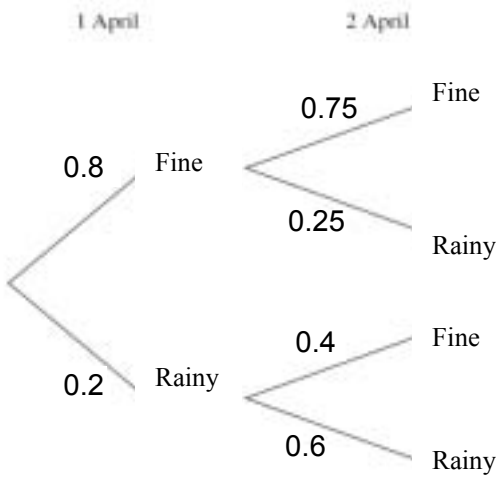
Question	Answer	Marks	Guidance
1(a)	$1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2+pq^3+ pq^4$ $(+ pq^5)$ $0 < p < 1$, $p + q = 1$,
	0·598, $\frac{4651}{7776}$	**A1**	
		2	
1(b)	(1 – P(0, 1, 2)) $1-\left(\left(\frac{5}{6}\right)^{10}+{}^{10}\text{C}_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9+{}^{10}\text{C}_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8\right)$	**M1**	${}^{10}\text{C}_x\ p^x\left(1-p\right)^{10-x}$, $0 < p < 1$, any p, $x \neq 0,10$
	1 – (0·1615056 + 0·3230111 + 0·290710)	**A1**	Correct expression, accept unsimplified, condone omission of final bracket
	0·225	**A1**	$0{\cdot}2247 < p \leq 0{\cdot}225$, WWW
		3	

Question	Answer	Marks	Guidance
2(a)	P(1 red) = $\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3$	**M1**	$\frac{a}{8}\times\frac{b}{7}\times\frac{c}{6}\times k$ or $\frac{5}{d}\times\frac{3}{e}\times\frac{2}{f}\times 3$, $1\leqslant a,b,c \leqslant 5$, $d, e, f \leqslant 8$, *a, b, c, d, e, f, k* all integers. $1 < k \leqslant 3$,
	$\frac{15}{56}$	**A1**	AG, WWW
	Alternative method for question 2(a)		
	$\frac{{}^5C_1\times{}^3C_2}{{}^8C_3}$	**M1**	$\frac{{}^aC_1\times{}^bC_2}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e\left(or\ {}^aC_1\times{}^bC_2\right)}{{}^5C_3\times{}^3C_0+{}^5C_2\times{}^3C_1+{}^5C_1\times{}^3C_2+{}^5C_0\times{}^3C_3}$, $a+b=8$, $d+e=3$
	$\frac{15}{56}$	**A1**	AG, WWW, $\frac{15}{56}$ must be seen
		2	
2(b)	x: 0, 1, 2, 3 Prob.: $\frac{1}{56}$, $\frac{15}{56}$, $\frac{30}{56}=\frac{15}{28}$, $\frac{10}{56}=\frac{5}{28}$ 0·0179, 0·268, 0·536, 0·179	**B1**	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		B1	2 of P(0), P(2) and P(3) correct
		B1 FT	4th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct
		3	

Question	Answer	Marks	Guidance
2(c)	$\text{Var}(X) = \dfrac{(0^2\times1)+1^2\times15+2^2\times30+3^2\times10}{56} - \left(\dfrac{15}{8}\right)^2$ $= \dfrac{15}{56}+\dfrac{120}{56}+\dfrac{90}{56}-\left(\dfrac{15}{8}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have ' – mean^2' (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\dfrac{225}{448}$, 0·502	**A1**	
		2	

Question	Answer	Marks	Guidance
3(a)	$P(X > 11.3) = P(z > \dfrac{11.3-10.1}{1.3}) = P(z > 0\cdot9231)$	**M1**	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 – 0.822	**M1**	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	0·178	**A1**	0.1779…
		3	
3(b)	$z = -0\cdot674$	**B1**	±0.674 seen (critical value)
	$\dfrac{t-10.1}{1.3} = -0\cdot674$	**M1**	An equation using ±standardisation formula with a z-value, condone $\sqrt{\sigma}$ or σ^2, continuity correction.
	$t = 9\cdot22$	**A1**	AWRT. Only dependent on M1
		3	

Question	Answer	Marks	Guidance
3(c)	P(8.9 < X < 11.3) = 1 – 2 × *their* **3(a)** ≡ 2(1 – *their* **3(a)**) – 1 ≡ 2(0·5 – *their* **3(a)**) =0.644	**B1 FT**	FT from *their* **3(a)** < 0·5 or correct, accept unevaluated probability OE
	Number of days = 90 × 0·644 = 57·96	**M1**	90 × *their p* seen, 0 < p < 1
	So 57 (days)	**A1 FT**	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
	Alternative method for question 3(c)		
	$\mathrm{P}\left(\frac{8\cdot9-10\cdot1}{1\cdot3}<z<\frac{11.3-10.1}{1\cdot3}\right)$ $=\Phi(0\cdot9231)-(1-\Phi(0\cdot9231))$ oe = 0·822 – (1 – 0·822) = 0·644	**B1**	Accept unevaluated probability
	Number of days = 90 × 0·644 = 57·96	**M1**	90 × *their p* seen, 0 < p < 1
	So 57 (days)	**A1 FT**	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
		3	

Question	Answer	Marks	Guidance
4(a)		**B1**	All probabilities correct, may be on branch or next to 'Fine/Rainy' Ignore additional branches.
		1	
4(b)	$0{\cdot}8 \times 0{\cdot}75 + 0{\cdot}2 \times 0{\cdot}4$ $(= 0{\cdot}6 + 0{\cdot}08)$	**M1**	Correct or FT from *their* diagram unsimplified, all probabilities $0 < p < 1$. Partial evaluation only sufficient when correct. Accept working in **4(b)** or by the tree diagram.
	$0.68, \frac{17}{25}$	**A1**	From supporting working
		2	

Question	Answer	Marks	Guidance
4(c)	$0{\cdot}8 \times 0{\cdot}75 \times 0{\cdot}25 + 0{\cdot}8 \times 0{\cdot}25 \times 0{\cdot}6$	**M1**	$a \times b \times c + a \times 1\text{-}b \times d$, $0 < c,\ d \leqslant 1$, *a, b* consistent with *their* tree diagram or correct, no additional terms
	$0{\cdot}15 + 0{\cdot}12$	**A1**	At least one term correct, accept unsimplified
	0.27	**A1**	Final answer
		3	
4(d)	P(Y) = *their* **(c)** $+ 0{\cdot}2 \times 0{\cdot}4 \times 0{\cdot}25 + 0{\cdot}2 \times 0{\cdot}6 \times 0{\cdot}6$ $(= 0{\cdot}362)$	**B1 FT**	*their* **(c)** $+ e \times f \times g + e \times (1\text{-}f) \times h$, $0 < g,\ h \leqslant 1$, *e, f* consistent with *their* tree diagram, or correct
	$\mathrm{P(X\|Y)} = \dfrac{\textit{their}\,(\mathrm{c})}{\textit{their}\,\mathrm{P(Y)}} = \dfrac{0.27}{0.362}$	**M1**	*their* **4(c)** (or correct)/*their* previously calculated and identified P(Y) or a denominator involving 3 or 4 3-factor probability terms consistent with *their* tree diagram & third factor $0 < p < 1$
	$0{\cdot}746,\ \dfrac{373}{500}$ or $\dfrac{135}{181}$	**A1**	(0.7458…)
		3	

Question	Answer	Marks	Guidance
5(a)	*Dados* | | *Linva* 8 6 | 0 | 0 2 9 6 5 2 0 0 | 1 | 0 1 2 5 6 8 2 | 2 | 6 | 3 | 2 6 2 | 4 | 0 KEY 6| 3| 2 means 36 cm (snow) in Dados and 32 cm (snow) in Linva	**B1**	Correct stem can be upside down, ignore extra values
		B1	Correct Dados labelled, leaves in order and lined up vertically (less than midway to next column), no commas etc, no extra terms
		B1	Correct Linva on opposite side of stem labelled, leaves in order and lined up vertically (less than midway to next column), no commas etc, no extra terms
		B1	Correct single key for their diagram, need both resorts identified and 'cm' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SCB1** if both keys meet these criteria B0B1B0SCB1 max.
		4	
5(b)	Median or Q2 = 15 (cm)	**B1**	Correct
	UQ or Q3 = 28 cm, LQ or Q1 = 10 cm IQR = 28 – 10	**M1**	$22 \leqslant UQ \leqslant 36 - 8 \leqslant LQ \leqslant 10$
	18 (cm)	**A1**	WWW
		3	
5(c)	On average the snowfall in Davos is higher	**B1 FT**	FT from *their* **5(b)** values for Dados. Statement comparing central tendency in context
	The amount of snowfall in Linva varies more than in Davos	**B1 FT**	Statement comparing spread in context Note: simply stating and comparing the values is not sufficient.
		2	

Question	Answer	Marks	Guidance
6(a)	${}^9C_6$ ($\times$ ${}^3C_3$)	**M1**	${}^9C_k \times n$, k = 6, 3, n = 1,2 oe Condone ${}^9C_6 + {}^3C_3$, ${}^9P_6 \times {}^3P_3$
	84	**A1**	Accept unevaluated.
		2	
6(b)	Number with 3 Baker children = ${}^6C_2$ or 15	**B1**	Correct seen anywhere, not multiplied or added
	Total no of selections = ${}^9C_5$ or 126 Probability = $\dfrac{\text{number of selections with3 Baker children}}{\text{total number of selections}}$	**M1**	Seen as denominator of fraction
	$\dfrac{15}{126}$, 0·119	**A1**	OE, e.g. $\dfrac{5}{42}$
	Alternative method for question 6(b)		
	$\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\left(\times\dfrac{6}{6}\right)\left(\times\dfrac{5}{5}\right)\times {}^5C_3$	**B1**	${}^5C_3$ (OE) or 10 seen anywhere, multiplied by fractions only, not added
		M1	$\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\left(\times\dfrac{6}{6}\right)\left(\times\dfrac{5}{5}\right)\times k$, $1 \leqslant k$, k integer
	$\dfrac{15}{126}$, 0·119	**A1**	OE, e.g. $\dfrac{5}{42}$
		3	

Question	Answer	Marks	Guidance
6(c)	[Total no of arrangements = 9!] [Arrangements with men together = 8! × 2] Not together: 9! –	**M1**	9! – *k* or 362880 – *k*, *k* an integer<362 880
	8! × 2	**B1**	8! × 2(!) or 80 640 seen anywhere
	282 240	**A1**	Exact value
	Alternative method for question 6(c)		
	7! × 8 × 7	**B1**	7! × *k*, *k* positive integer > 1
		M1	$m \times 8 \times 7$, $m \times {}^{8}P_{2}$, $m \times {}^{8}C_{2}$ *m* positive integer > 1
	282 240	**A1**	Exact value
		3	
6(d)	7! × 2 × 7	**M1**	7! × *k*, *k* positive integer > 1 If 7! not seen, condone 7 × 6 × 5 × 4 × 3 × 2 × (1) × *k* *or* 7 × 6! × *k* only
		M1	*m* × 2 × 7, *m* positive integer > 1
	70 560	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
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GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
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6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$P(56 < X < 66) = P\left(\frac{56-62}{5} < z < \frac{66-62}{5}\right)$ $= P(-1.2 < z < 0.8)$	**M1**	Using ± standardisation formula at least once, no $\sqrt{\sigma}$ or σ^2, allow continuity correction
	$\Phi(0.8) + \Phi(1.2) - 1$ $= 0.7881 + 0.8849 - 1$	**M1**	Appropriate area Φ, from standardisation formula in final solution
	0.673	**A1**	
		3	
1(b)	$z = 1.127$	**B1**	±(1.126 – 1.127) seen, 4 sf or more
	$\frac{60t-62}{5} = 1.127$ $60t = 5.635+62=67.635$	**M1**	z-value = $\pm\frac{(60t-62)}{5}$ condone z-value = $\pm\frac{(t-62)}{5}$ no continuity correction, condone $\sqrt{\sigma}$ or σ^2
	$t = 1.13$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
2(a)	$\left(\frac{5}{6}\right)^8$	**M1**	p^8, $0 < p < 1$, no x, + or -
	0.233	**A1**	
		2	
2(b)	36	**B1**	
		1	
2(c)	$P(X=10) + P(X=11) = \left(\frac{35}{36}\right)^9 \frac{1}{36} + \left(\frac{35}{36}\right)^{10} \frac{1}{36}$	**M1**	OE, unsimplified expression in form $p^9q + p^{10}q$, $p + q = 1$, no $\times$
	0.0425	**A1**	
		2	

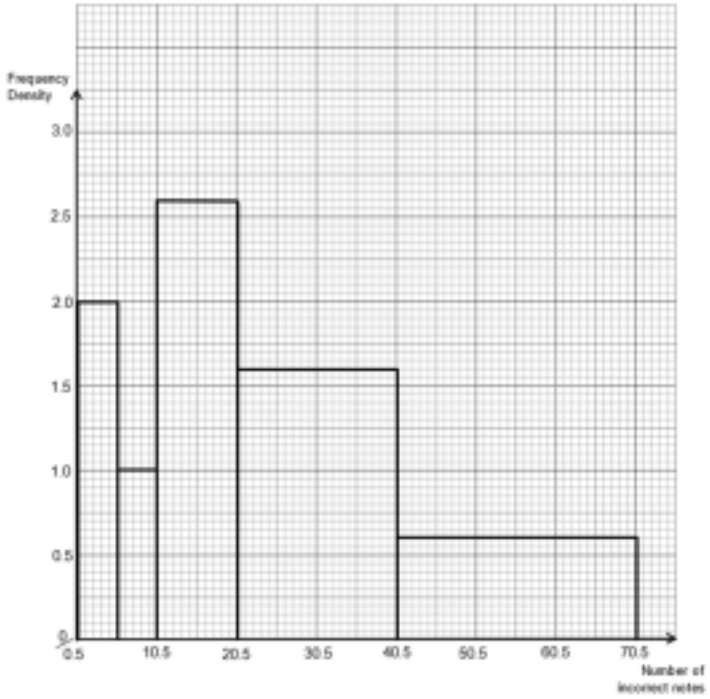
Question	Answer	Marks	Guidance
3(a)	Scenarios: 6W 0M ${}^{9}C_{6} = 84$ 5W 1M ${}^{9}C_{5} \times {}^{5}C_{1} = 126 \times 5 = 630$ 4W 2M ${}^{9}C_{4} \times {}^{5}C_{2} = 126 \times 10 = 1260$	**M1**	Correct number of ways for either 5 or 4 women, accept unsimplified
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios.
	Total = 1974	**A1**	
		3	
3(b)	Total number of ways = ${}^{14}C_{6}$ (3003) Number with sister and brother = ${}^{12}C_{4}$ (495) Number required = ${}^{14}C_{6}$ –	**M1**	${}^{14}C_{6}$ – a value
	${}^{12}C_{4} = 3003 - 495$	**M1**	${}^{12}C_{x}$ or ${}^{n}C_{4}$ seen on its own or subtracted from *their* total, $x \leqslant 6$, $n \leqslant 13$
	2508	**A1**	
	Alternative method for question 3(b)		
	Number of ways with neither = ${}^{12}C_{6} = 924$	**M1**	${}^{12}C_{6}$ + a value
	Number of ways with either brother or sister (not both) = ${}^{12}C_{5} \times 2$ (= 792×2) = 1584	**M1**	${}^{12}C_{x} \times 2$ or ${}^{n}C_{5} \times 2$ seen on its own or added to *their* number of ways with neither, $x \leqslant 5$, $n \leqslant 12$
	Number required = 924 + 1584 = 2508	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$0.65^7 + {}^7C_1\ 0.65^6\ 0.35^1 + {}^7C_2\ 0.65^5\ 0.35^2$	**M1**	Binomial term of form ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1$, any p, $x \neq 0, 7$
	0.049022 + 0.184776 + 0.29848	**A1**	Correct unsimplified answer
	0.532	**A1**	
		3	
4(b)	Mean = $142 \times 0.35 = 49.7$ Variance = $142 \times 0.35 \times 0.65 = 32.305$	**B1**	Correct unsimplified *np* and *npq* (condone $\sigma = 5.684$ evaluated)
	$P(X > 40) = P(z > \frac{40.5 - 49.7}{\sqrt{32.305}})$	**M1**	Substituting *their* μ *and* σ (no $\sqrt{\sigma}$ or σ^2) into ±standardisation formula with a numerical value for '40.5'
	$P(z > -1.619)$	**M1**	Using either 40.5 or 39.5 within a ±standardisation formula
		M1	Appropriate area Φ, from standardisation formula $P(z > \ldots)$ in final solution, must be probability
	0.947	**A1**	Correct final answer
		5	

Question	Answer	Marks	Guidance
5(a)	Total number of ways = $\frac{8!}{3!2!}$ (= 3360)	**B1**	Correct unsimplified expression for total number of ways
	Number of ways with V and E in correct positions = $\frac{6!}{2!\times 2!}$ (= 180)	**B1**	$\frac{6!}{2!\times 2!}$ alone or as numerator in an attempt to find the number of ways with V and E in correct positions. No $\times, \pm$
	Probability = $\frac{180}{3360}$ $\left(=\frac{3}{56}\right)$ or 0.0536	**B1 FT**	Final answer from *their* $\frac{6!}{2!\times 2!}$ divided by *their* total number of ways
	Alternative method for question 5(a)		
	$\frac{1}{8}\times\frac{3}{7}$	**M1**	$\frac{a}{8}\times\frac{b}{7}$ seen, no other terms (correct denominators)
		M1	$\frac{1}{c}\times\frac{3}{d}$ seen, no other terms (correct numerators)
	$\frac{3}{56}$ or 0.0536	**A1**	
		3	

Question	Answer	Marks	Guidance
5(b)	Rs together and Es together: 5! (120)	**B1**	Alone or as numerator of probability to represent the number of ways with Rs and Es together, no ×, +, –
	Es together: $\frac{6!}{2!}(=360)$	**B1**	Alone or as denominator of probability to represent the number of ways with Es together, no ×, + or –
	Probability = $\dfrac{\frac{5!}{6!}}{\frac{2!}{}}$	**M1**	$\dfrac{\textit{their}\,5!}{\textit{their}\,\frac{6!}{2!}}$ seen
	$\frac{1}{3}$	**A1**	OE
	Alternative method for question 5(b)		
	P(Rs together and Es together): $\dfrac{5!}{\textit{their}\text{ total number of ways}}\left(=\frac{1}{28}\right)$	**B1**	
	P(Es together): $\dfrac{\frac{6!}{2!}}{\textit{their}\text{ total number of ways}}\left(=\frac{3}{28}\right)$	**B1**	Alone or as numerator of probability to represent the P(Rs and Es together), no ×, +, –
	Probability = $\dfrac{\frac{1}{28}}{\frac{3}{28}}$	**M1**	Alone or as denominator of probability to represent the P(Es together), no ×, + or –
	$\frac{1}{3}$	**A1**	OE, $\dfrac{\textit{their}\,\frac{1}{28}}{\textit{their}\,\frac{3}{28}}$ seen
		4	

Question	Answer	Marks	Guidance
6(a)	Scenarios: HHT: $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{5}=\frac{4}{45}$ HTH: $\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}=\frac{8}{45}$ THH: $\frac{1}{3}\times\frac{2}{3}\times\frac{4}{5}=\frac{8}{45}$	**M1**	One 3 factor probability with 3, 3, 5 as denominators
		M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios
	Total = $\frac{20}{45}=\frac{4}{9}$	**A1**	AG, Total of 3 products with clear context
		3	
6(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob.</td><td>$\frac{1}{45}$</td><td>$\frac{8}{45}$</td><td>$\frac{20}{45}$</td><td>$\frac{16}{45}$</td></tr></table>	**B1**	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
		B1	2 of P(0), P(1) and P(3) correct
		B1 FT	3 or 4 probabilities sum to 1 with P(2) correct
		3	
6(c)	$\text{Var(X)} = \frac{0^2\times1+1^2\times8+2^2\times20+3^2\times16}{45}-\left(\frac{32}{15}\right)^2$ $= \frac{8}{45}+\frac{80}{45}+\frac{144}{45}-\left(\frac{32}{15}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have '– mean^2' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values
	$\frac{136}{225}$ or 0.604	**A1**	
		2	

Question	Answer	Marks	Guidance
7(a)	Class widths: 5, 5, 10, 20, 30 Frequency density: 2, 1, 2.6, 1.6, 0.6	**M1**	At least 3 class widths correct and used in a calculation
		M1	At least 3 correct frequency densities unsimplified – FT *their* class widths
		**A1**	All correct heights on a histogram using a linear vertical scale from zero – no FT
		B1	Correct upper bar ends (5.5, 10.5, 20.5, 40.5, 70.5) and 4 correct lower bar ends of 5.5, 10.5, 20.5, 40.5. Condone 0 or 1.
		B1	Linear scales with at least 3 values indicated on each axis, vertical scale from 0, axes labelled 'fd' and 'no. of (incorrect) notes', or better.
		5	
7(b)	LQ: 11 – 20 UQ: 21 – 40	**B1**	Both UQ and LQ correct
	Greatest IQR = 40 – 11 = 29	**B1 FT**	Subtract lower end of *their* LQ interval from upper end of *their* UQ interval
		2	

Question	Answer	Marks	Guidance
7(c)	Midpoints: 3 8 15.5 30.5 55.5	**M1**	At least 4 midpoints correct and used
	Mean $= \frac{3\times10+8\times5+15.5\times26+30.5\times32+55.5\times18}{91}$ $= \frac{30+40+403+976+999}{91}$ $= \frac{2448}{91}$	**M1**	Correct formula with *their* midpoints (not upper boundary, lower boundary, class width, frequency density, frequency or cumulative frequency)
	$26.9,\ 26\frac{82}{91}$	**A1**	Accept 26 or 27
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left(\frac{3}{4}\right)^6 \frac{1}{4}$	**M1**	$(1-p)^6\, p,\ 0<p<1$
	$0.0445,\ \frac{729}{16384}$	**A1**	
		2	
1(b)	$\left(\frac{3}{4}\right)^9$	**M1**	$\left(\frac{3}{4}\right)^n$ *or* $p^n,\ 0<p<1, n=8,9,10$
	$0.0751,\ \frac{19683}{262144}$	**A1**	
		2	

Question	Answer	Marks	Guidance
2(a)	$\left[\frac{\Sigma x}{40}-k=\frac{\Sigma(x-k)}{40}\right]$ $\frac{40\times 34}{40}-k=\frac{520}{40}$	**M1**	Forms an equation involving Σx, $\Sigma(x\text{-}k)$ and k. Accept at a numeric stage with k.
	$k\,[=34-13]=21$	**A1**	Evaluated.
		2	

Question	Answer	Marks	Guidance
2(b)	$\text{Var} = \left[\frac{\Sigma(x-k)^2}{40} - \left(\frac{\Sigma(x-k)}{40}\right)^2\right] = \frac{9640}{40} - \left(\frac{520}{40}\right)^2 = [241 - 13^2 =]$	**M1**	Values substituted into an appropriate variance formula, accept unsimplified.
	72	**A1**	
		2	

Question	Answer	Marks	Guidance
3	$\left[\text{P}(T\|B') = \frac{\text{P}(T \cap B')}{\text{P}(B')}\right]$ $\text{P}(B') = 0.45\times0.7 + 0.35\times0.4 + 0.2\times1$ $\left[= 0.655, \frac{131}{200}\right]$	**M1**	$0.45\times a + 0.35\times b + 0.2[\times1], a = 0.7, 0.3b = 0.4, 0.6$, seen anywhere.
		A1	Correct, accept unsimplified.
	$\text{P}(T \cap B') = 0.35\times0.4$ [= 0.14, $\frac{7}{50}$]	**M1**	Seen as numerator or denominator of a fraction.
	$\text{P}(T \mid B') = \frac{\textit{their}\,0.14}{\textit{their}\,0.655}$	**M1**	Values substituted into conditional probability formula correctly. Accept unsimplified. Denominator sum of 3 two-factor probabilities (condone omission of 1 from final factor). If clearly identified, condone from incomplete denominator.
	0.214, $\frac{28}{131}$	**A1**	If 0 marks awarded, **SC B1** 0.214 WWW.
		5	

Question	Answer	Marks	Guidance
4(a)	x: -1, 0, 1, 2, 3 p: $\frac{1}{12} = 0.0833$, $\frac{2}{12} = 0.167$, $\frac{4}{12} = 0.333$, $\frac{3}{12} = 0.25$, $\frac{2}{12} = 0.167$	**B1**	<table><tr><td></td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>– 1</td><td>–1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>3</td></tr></table> Table with x values and at least one probability substituted, $0 < p < 1$. Condone any additional x values if probability stated as 0.
		B1	2 correct identified probabilities.
		B1	All probabilities correct (accept to 3sf). **SC** if less than 2 correct probabilities: **SC B1** 4 or 5 probabilities summing to one.
		3	
4(b)	$\mathrm{E}(X) = -\frac{1}{12}+\frac{4}{12}+\frac{6}{12}+\frac{6}{12}\left[=\frac{15}{12}\right]$	**M1**	May be implied by use in Variance, accept unsimplified expression. Probabilities must sum to 1 ± 0.001.
	$\mathrm{Var}(X) = \frac{1}{12}+0+\frac{4}{12}+\frac{12}{12}+\frac{18}{12}-\left(\frac{15}{12}\right)^2$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$. **FT** accept probabilities not summing to 1. Condone $\frac{35}{12}-\left(\frac{15}{12}\right)^2$ *or* $\frac{35}{12}-\frac{25}{9}$ from correct table.
	$\left[\frac{35}{12}-\frac{25}{16}=\right]\frac{65}{48}$, 1.35	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
5(a)	[8! =] 40 320	**B1**	Evaluated, exact value only.
		1	
5(b)	**Method 1** [^ ^ ^ R ^ ^ S ^ ^]		
	$7! \times {}^8C_2 \times 2$	**M1**	$7! \times k$ seen, k an integer > 1.
		M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 7, 8 or 9, m an integer > 1.
	282 240	**A1**	Exact value only. **SC B1** for final answer 282 240 WWW.
	Method 2 [Total number of arrangements – Arrangements with R & S together]		
	$9! - 8! \times 2$	**M1**	$9! - k$, k an integer < 362 880 .
		M1	$m - 8! \times n$, m an integer > 40 320, n = 1,2.
	282 240	**A1**	Exact value only. **SC B1** for final answer 282 240 WWW.
		3	
5(c)	${}^9C_5$ [$\times {}^4C_4$]	**M1**	${}^9C_x$ [$\times {}^{9-x}C_{9-x}$,] x = 4, 5. Condone × 1 for ${}^{9-x}C_{9-x}$. Condone use of P.
	126	**A1**	WWW
		2	

Question	Answer	Marks	Guidance
5(d)	[Number of ways with Raman and Sanjay together on back row =] $^7C_3$ [Number of ways with Raman and Sanjay together on front row =] $^7C_2$	**M1**	^7C_x seen, $x = 3$ or 2.
	[Total =] 35 + 21	**M1**	Summing two correct scenarios.
	56	**A1**	Evaluated – may be seen used in probability. If **M0** scored, **SC B1** for 56 WWW.
	Probability = $\frac{their\,56}{their(c)} = \frac{56}{126}, \frac{4}{9}$, 0.444	**B1 FT**	FT *their* 56 from adding 2 or more scenarios in numerator and *their* **(c)** or correct as denominator.
		4	

Question	Answer	Marks	Guidance
6(a)	Rebels | Sharks 6 | 6 8 9 8 5 | 7 | 1 2 4 5 5 6 8 9 6 5 4 3 2 2 0 | 8 | 3 3 4 5 6 9 5 3 | 9 | 2 2 | 10 | Key: 8 | 7 | 2 means 78 kg for Rebels and 72 kg for Sharks	**B1**	Correct stem, ignore extra values (not in reverse).
		B1	Correct Rebels labelled on left, leaves in order from right to left and lined up vertically, no commas.
		B1	Correct Sharks labelled on same diagram, leaves in order and lined up vertically, no commas.
		B1	Correct key for their diagram, need both teams identified and 'kg' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SC B1** if both keys meet these criteria.
		4	

Question	Answer	Marks	Guidance
6(b)	Median = 84 (kg)	**B1**	
	[UQ = 93, LQ = 80] 93 – 80	**M1**	$95 \leqslant UQ \leqslant 89 - 79 \leqslant LQ \leqslant 82$
	[IQR =] 13 (kg)	**A1**	WWW
		3	
6(c)	Box and whisker with end points 75 and 102	**B1**	Whiskers drawn to correct end points not through box, not joining at top or bottom of box.
	Median and quartiles plotted as found in **(b)**	**B1 FT**	Quartiles and median plotted as box graph.
		2	
6(d)	e.g. Average weight of Rebels is higher than average weight of Sharks	**B1**	Acceptable answers refer to: Range, skew, central tendency within context. E.g. range of Rebels is greater **B0**. Range of weights of the rebels is greater **B1**. Simple value comparison insufficient.
		1	

Question	Answer	Marks	Guidance
7(a)(i)	$P(X > 142) = P\left(Z > \frac{142-125}{24}\right)$	**M1**	Substitution of correct values into the ±Standardisation formula, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	[= P($Z > 0.7083$) =]1 – 0.7604	**M1**	Appropriate numerical area Φ, from final process, must be probability, expect $p < 0.5$.
	0.2396	**A1**	$0.239 \leqslant p \leqslant 0.240$ to at least 3sf.
	Their 0.2396 × 365 [= 87.454]	**M1**	**FT** *their* 4sf (or better) probability.
	87 or 88	**A1 FT**	Final answer must be positive integer, no indication of approximation/rounding, only dependent on previous **M** mark. **SC B1 FT** for *their* 3sf probability × 365 = integer value, condone 0.24 used.
		5	
7(a)(ii)	$P(0, 1) = 0.7604^{10} + {}^{10}C_1 \times 0.2396^1 \times 0.7604^9$ [= 0.064628 + 0.20364]	**M1**	One term: ${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1 FT	Correct unsimplified expression using *their* probability to at least 3sf from **(a)(i)** or correct.
	0.268	**A1**	AWRT, WWW.
		3	
7(b)	$z = \pm 1.282$	**B1**	Correct value only, critical value.
	$\frac{t-125}{24} = -1.282$	**M1**	Use of ± Standardisation formula with correct values substituted, allow continuity correction, σ^2, $\sqrt{\sigma}$, to form an equation with a z-value and not probability.
	$t = 94.2$	**A1**	AWRT, condone AWRT 94.3. Not dependent on **B** mark.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\frac{82}{180}, \frac{41}{90}$, 0.456	B1	
		1	
1(b)	$\left[\mathrm{P(M\|D)} = \frac{P(M \cap D)}{P(D)}\right] = \frac{\frac{11}{180}}{\frac{20}{180}+\frac{11}{180}}$ or $\frac{0.6011}{0.1722}$	M1	*Their* identified $\frac{P(M \cap D)}{P(D)}$ *or* from data table $\frac{11}{20+11}$, accept unsimplified, condone × 180.
	$\frac{11}{31}$, 0.355	A1	Final answer.
		2	

Question	Answer	Marks	Guidance
1(c)	$P(F) = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE $P(G) = \frac{82}{180}, \frac{41}{90} 0.4556$ OE $P(F \cap G) = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $P(F) \times P(G) = \frac{100}{180} \times \frac{82}{180} = \frac{41}{162}, 0.2531$ OE $\left[\neq \frac{38}{180} \right]$ Not independent	**M1**	*Their* identified P(F) × *their* identified P(G) or correct seen, can be unsimplified.
		A1	$\frac{41}{162}, \frac{38}{180}, P(F \cap G)$ and $P(F) \times P(G)$ seen with correct conclusion, WWW. Values and labels must be seen.
	Alternative method for question 1(c)		
	$P(F \cap G) = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $P(G) = \frac{82}{180}, \frac{41}{90}, 0.4556$ OE $P(F\|G) = \frac{\frac{38}{180}}{\frac{82}{180}} = \frac{19}{41}, 0.4634$ OE $\neq P(F) = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE Not independent	**M1**	P(F|G) (OE) unsimplified with *their* identified probs or correct
		A1	$\frac{19}{41}, \frac{100}{180}, P(F \cap G)$ and $P(F\|G)$ seen with correct conclusion WWW. Values and labels must be seen.
		2	

Question	Answer	Marks	Guidance
2(a)	$^{11}C_5 \times {}^4C_1$	**M1**	$^{11}C_5 \times {}^4C_1$ condone $^{11}P_5 \times {}^4P_1$ no +, –, × or ÷.
	1848	**A1**	CAO as exact.
		2	
2(b)	**Method 1 [**Identifying scenarios]		
	[Neither selected =] $^{13}C_6$ [= 1716] [Only Jane selected =] $^{13}C_5$ [= 1287] [Only Kate selected =] $^{13}C_5$ [= 1287]	**M1**	Either $^{13}C_6$ seen alone or $^{13}C_5$ seen alone or × 2 (condone $^{13}P_n$, n = 5,6).
	[Total =] 1716 + 1287 + 1287	**M1**	Three correct scenarios only added, accept unsimplified (values may be incorrect).
	4290	**A1**	
	Method 2 [Total number of selections – selections with Jane and Kate both picked]		
	$^{15}C_6$ - $^{13}C_4$ [= 5005 – 715]	**M1**	$^{15}C_6 – k$, k a positive integer < 5005, condone $^{15}P_6$.
		M1	$m – {}^{13}C_4$, m integer > 715, condone $n – {}^{13}P_4$, n > 17 160.
	4290	**A1**	
		3	
			SC Where the condition of **2(a)** is also applied in **2(b)**, the final answer is 1512 **SC M1 M1 A0** max. The method marks can be earned for the equivalent stages in each method. Method 1 $^4C_1 \times {}^9C_5 + {}^4C_1 \times {}^9C_4 \times 2$ Method 2 $^4C_1 \times {}^{11}C_5 – {}^4C_1 \times {}^9C_3$

Question	Answer	Marks	Guidance
3(a)	For one yellow: YGG + GYG +GGY $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times 3$	**M1**	$\frac{a}{9}\times\frac{b}{8}\times\frac{c}{7}$, $0 < a,b,c$ integers $\leqslant 5$, for one arrangement.
		M1	*Their* three-factor probability $\times 3$, ${}^3C_1$, ${}^3C_2$ or ${}^3P_1$, (or repeated adding) no additional terms.
	$\left[\frac{180}{504}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.
		3	
	Alternative method for question 3(a)		
	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_3}$	**M1**	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_r}$, $r = 2, 3, 4$
		M1	$\frac{{}^5C_s\times{}^4C_t}{{}^9C_3}$, $s + t = 3$
	$\left[\frac{30}{84}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, WWW.
		3	

Question	Answer	Marks	Guidance
3(b)	X: 0 | 1 | 2 | 3 $P(X)$: $\frac{24}{504}$ $\left[=\frac{1}{21}, 0.0476\right]$ | $\frac{180}{504}$ $\left[=\frac{5}{14}, 0.357\right]$ | $\frac{240}{504}$ $\left[=\frac{10}{21}, 0.476\right]$ | $\frac{60}{504}$ $\left[=\frac{5}{42}, 0.119\right]$	**B1**	Table with correct X values and one correct probability inserted appropriately. Condone any additional X values if probability stated as 0.
		B1	Second identified correct probability, may not be in table.
		B1	All probabilities identified and correct . **SC** if less than 2 correct probabilities or X value(s) omitted: **SC B1** 3 or 4 probabilities summing to one.
		3	
3(c)	[E(X) =] $\frac{840}{504}, \frac{5}{3}$, 1.67	**B1**	OE Must be evaluated. **SC B1 FT** correct unsimplified expression from incorrect **3(b)** using at least 3 probabilities, $0 < p < 1$.
		1	

Question	Answer	Marks	Guidance
4(a)	$\frac{9!}{3!}$	**M1**	$\frac{9!}{e!}$, $e = 2, 3$
	60 480	**A1**	
		2	

Question	Answer	Marks	Guidance
4(b)	$\frac{7!}{3!}\times 2\times 6$	**M1**	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times 2\times q$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 8$ all integers.
		M1	$\frac{m!}{n!}\times p\times 6$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant p \leqslant 2$ all integers. (Accept 3P2 for 6) If **M0 M0 M0** awarded, **SC M1** for $t \times 12$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	**A1**	Exact value.
	Alternative method for question 4(b)		
	$\frac{{}^7\mathrm{P}_2\times 6!\times 2}{3!}$	**M1**	$\frac{6!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times {}^7\mathrm{P}_2\times q$ $m = 6,9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 2$ all integers.
		M1	$\frac{m!}{n!}\times {}^7\mathrm{P}_r\times 2$ $m = 6, 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant r \leqslant 5$ all integers. If **M0 M0 M0** awarded, **SC M1** for $t \times 84$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	**A1**	Exact value.

Question	Answer	Marks	Guidance
4(b)	**Alternative method for question 4(b)**		
	$\frac{7!}{3!} \times 4P2$	**M1**	$\frac{7!}{3!} \times k$ seen, k an integer > 0.
		M1	$t \times {}^4P_2$ or 12, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
		M1	$\frac{m!}{n!} \times 4P2$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$ all integers.
	10 008	**A1**	Exact value.
		4	

Question	Answer	Marks	Guidance
5(a)	[P(0, 1, 2) =] ${}^{10}C_0\, 0.16^0\, 0.84^{10} + {}^{10}C_1\, 0.16^1\, 0.84^9 + {}^{10}C_2\, 0.16^2\, 0.84^8$ [= 0.17490 + 0.333145 + 0.28555]	**M1**	One term: ${}^{10}C_x p^x\, (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	**A1**	$0.7935 < p \leqslant 0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
5(b)	$(0.84)^7\, 0.16$	**M1**	$(1-p)^7 p$, $0 < p < 1$
	0.0472	**A1**	0.0472144 to at least 3sf.
		2	

Question	Answer	Marks	Guidance
5(c)	$4 \times 0.0472 \times (1 - 0.0472)^3$	**M1**	$4 \times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	**A1**	$0.163 \leqslant p \leqslant 0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

Question	Answer	Marks	Guidance
6(a)	$\left[\mathrm{P}(X > 28.6) = \right] P\left(Z > \dfrac{28.6 - 32.2}{9.6}\right)$ $\left[= P(Z > -0.375)\right]$	**M1**	28.6, 32.2 and 9.6 substituted appropriately in ± Standardisation formula once, allow continuity correction of ± 0.05, no σ^2, $\sqrt{\sigma}$.
	$[\Phi(\textit{their}\, 0.375) =]$ *their* 0.6462	**M1**	Appropriate numerical area, from final process, must be probability, expect > 0.5.
	0.646	**A1**	AWRT
		3	
6(b)	$z = \pm 0.842$	**B1**	$0.841 < z \leqslant 0.842$ or $-0.842 \leqslant z < -0.841$ seen.
	$\dfrac{t - 32.2}{9.6} = 0.842$	**M1**	Substituting 32.2 and 9.6 into ± standardisation formula, no continuity correction, allow σ^2, $\sqrt{\sigma}$, must be equated to a z-value.
	$t = 40.3$	**A1**	$40.28 \leqslant t \leqslant 40.3$ WWW
		3	

Question	Answer	Marks	Guidance
6(c)	$P\left(-\frac{15}{9.6} < Z < \frac{15}{9.6}\right)$ $P(-1.5625 < Z < 1.5625)$	**M1**	Identifying at least one of $\frac{15}{9.6}$ and $-\frac{15}{9.6}$ as the appropriate z-values or substituting *their* (32.2 ± 15) into ± Standardisation formula once, no continuity correction, σ^2 nor $\sqrt{\sigma}$. Condone ±1.563 for **M1**.
	$[2\ \Phi(\frac{15}{9.6}) - 1\ ]$ $= 2 \times 0.9409 - 1$	**A1**	$p = 0.941$ AWRT SOI
		M1	Appropriate area $2\Phi - 1$ oe, (eg $1 - 2 \times 0.0591$, $2 \times (0.9409 - 0.5)$ or $0.9409 - 0.0591$), from final process, must be probability > 0.5.
	0.882	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Cumulative frequency graph drawn	**B1**	Axes labelled 'cumulative frequency' (or cf) from 0 to at least 140 and 'distance (or d) [in] m' from 0 to at least 1600, linear scales with at least 3 values stated.
		B1	All plotted correctly at correct upper end points (200 etc.) **curve** drawn accurately joined to (0, 0) (straight line segments B0) but no daylight above 140. Cf scale no less than 2 cm = 20 children .
		2	

Question	Answer	Marks	Guidance
7(b)	[UQ at 75% of 140 = 105, LQ at 25% of 140 = 35] [IQR:] 700 – 260	**M1**	Accept $660 \leqslant \text{UQ} \leqslant 720 - 240 \leqslant \text{LQ} \leqslant 290$. If values are outside our range, FT providing scales linear and increasing cf drawn.
	440	**A1**	Accept correct evaluation of $660 \leqslant$ *their* UQ $\leqslant 720 - 240 \leqslant$ *their* LQ $\leqslant 290$ with clear indication that graph has been used for at least one of 105 or 35.
		2	

Question	Answer	Marks	Guidance
7(c)	[Mean =] $\frac{16\times100+30\times250+42\times400+34\times700+12\times1050+6\times1400}{140}$	**B1**	<table><tr><td>Frequencies</td><td>16</td><td>30</td><td>42</td><td>34</td><td>12</td><td>6</td></tr><tr><td>Mid-points</td><td>100</td><td>250</td><td>400</td><td>700</td><td>1050</td><td>1400</td></tr></table> 5 or 6 correct frequency values seen.
		B1	5 or 6 correct midpoint values seen.
		M1	Values substituted into mean formula using *their* midpoints which must be in the class – condone 1 data error. Accept $\frac{1600+7500+16\,800+23\,800+12\,600+8400}{140}$ or $\frac{70\,700}{140}$. Condone $\frac{70\,770}{140}$ for **M1**.
	505	**A1**	WWW
	Variance = $\frac{16\times100^2+30\times250^2+42\times400^2+34\times700^2+12\times1050^2+6\times1400^2}{140}$ -505^2	**M1**	Values substituted into variance formula using (*their* mean)^2 and *their* midpoints and *their* frequencies (including for denominator). Accept unsimplified. Condone 1 data error. Accept: $[\frac{160\,000+1\,875\,000+6\,720\,000+16\,660\,000+13\,230\,000+11\,760\,000}{140}$ or $\frac{50\,405\,000}{140}$ or 360 035.7143] $-\left[505^2 \text{ or } 255\,025\right]$ If formula stated accept 105 010 or 105 011 WWW.
	S.d. = $\left[\sqrt{105\,010.7}=\right]324$	**A1**	WWW
		6	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

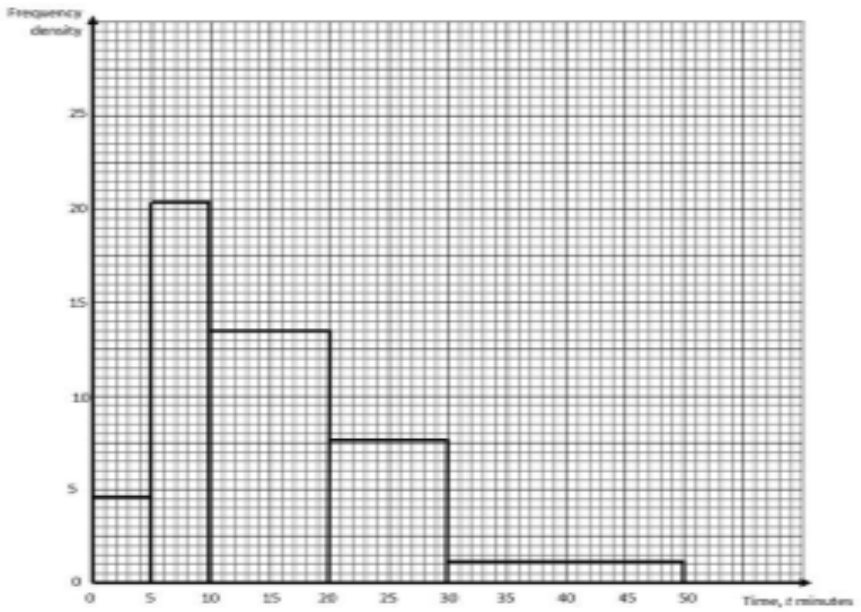
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$^{23}C_{17}$	**M1**	$^{23}C_x$ or $^{y}C_{17}$ or $^{z}C_6$, x, y or z are integers no +, –, × or ÷.
	100947	**A1**	CAO
		2	

Question	Answer	Marks	Guidance
2(a)	Lakeview | Riverside 9 4 0 | 1 | 8 8 8 7 6 2 | 2 | 0 1 3 4 5 5 3 2 0 | 3 | 0 6 7 1 | 4 Key: 6|2|3 means 26m for Lakeview and 23m for Riverside	**B1** **B1** **B1** **B1**	Correct stem, ignore extra values. Correct Lakeview labelled on left, leaves in order from right to left and lined up vertically, no commas. Correct Riverside labelled on same diagram, leaves in order and lined up vertically, no commas. Correct key for their diagram, need both teams identified and 'm' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn: **SC B1** if both keys meet these criteria.
		4	
2(b)	UQ = 32, LQ = 19	**M1**	$(30 \leqslant UQ \leqslant 33) - (14 \leqslant LQ \leqslant 22)$
	IQR = 32 – 19 = 13	**A1**	WWW
		2	

Question	Answer	Marks	Guidance
3(a)	Cw: 5 5 10 10 20	**M1**	At least 4 frequency densities calculated (f/cw), accept unsimplified and class widths ± 1 of true values. May be implied by graph.
	Fd: 4.6 20.4 13.5 7.6 1.2	**A1**	All heights correct on graph **NOT FT**
		**B1**	Bar ends at 0, 5, 10, 20, 30, 50 clear intention not to draw at 4.5 or 5.5 etc.
		B1	Axes labelled: Frequency density (fd), time (t) and mins (or appropriate title). Linear scales between 0 and 20.4 or above on vertical axis, and 0 and 50 or above on the horizontal axis. (Axes may be reversed.)
		4	
3(b)	$\frac{2.5\times 23+7.5\times 102+15\times 135+25\times 76+40\times 24}{360}$	**M1**	Uses at least 4 midpoint attempts (e.g. 2.5 ± 0.5) in correct formula, accept unsimplified expression, denominator either correct or *their* Σfrequencies .
	$\left[\frac{5707.5}{360}=\right]15.9,\ 15\frac{41}{48}$	**A1**	Evaluated.
		2	

Question	Answer	Marks	Guidance
4(a)	$P(X > 43.2) = P\left(Z > \frac{43.2 - 41.2}{3.6}\right) = P(Z > 0.5556)$	**M1**	Use of ±Standardisation formula once, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$1 - \Phi(0.5556) = 1 - 0.7108$	**M1**	Appropriate area Φ, from final process, must be probability.
	0.289	**A1**	AWRT
		3	
4(b)	Probability = 1 – *their* **(a)** = 1 – 0.2892 = 0.7108	**B1FT**	1 – *their* **(a)** or correct.
	$0.7108 \times 365 = 259.4$ 259, 260	**B1FT**	FT *their* 4SF (or better) probability, final answer must be positive integer.
		2	
4(c)	$z = \pm 1.645$	**B1**	CAO, critical z value.
	$\frac{t - 41.2}{3.6} = -1.645$	**M1**	Use of ±standardisation formula with μ, σ equated to a z-value, no continuity correction, allow σ^2, $\sqrt{\sigma}$.
	$t = 35.3$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	${}^5P_2 \times {}^7P_4$ **or** $5 \times 4 \times 7 \times 6 \times 5 \times 4$	**M1**	${}^5P_x \times {}^7P_y$, $1 \leqslant x \leqslant 4$, $1 \leqslant y \leqslant 6$
	16 800	**A1**	
		2	

Question	Answer	Marks	Guidance
5(b)	**Method 1 [**Identify scenarios]		
	With A and no 5: $8 \times {}^6P_4$ or $(1 \times 4 \times 6 \times 5 \times 4 \times 3) \times 2$ or 4C1 × 2! × 6P4 = 2880 With 5 and no A: ${}^4P_2 \times 4 \times {}^6P_3$ or $(4 \times 3 \times 1 \times 6 \times 5 \times 4) \times 4$ or 4P2 × 6C3 × 4! = 5760 With A and 5: $8 \times 4 \times {}^6P_3$ or $(4 \times 1 \times 1 \times 6 \times 5 \times 4) \times 8$ or 4C1 × 2! × 6C3 × 4! = 3840	**M1** **M1**	One number of ways correct, accept unsimplified. Add 2 or 3 identified correct scenarios only, accept unsimplified.
	[Total =] 12 480	**A1**	CAO
	Method 2 [total number of codes – number of codes with no A or 5]		
	No A or 5 : $(4 \times 3) \times (6 \times 5 \times 4 \times 3) = 4320$	**M1**	${}^4P_2 \times {}^6P_4$ or ${}^4C_2 \times {}^6C_4$ seen, accept unsimplified.
	Required number = *their* **(a)** – *their* 4320	**M1**	*Their* **5(a)** (or correct) – *their* (No A or 5) value.
	12 480	**A1**	
	Method 3 [subtracting **double counting**]		
	With A ${}^4P_1 \times {}^7P_4 \times 2$ or ${}^4C_1 \times 2 \times {}^7C_4 \times 4! = 6720$ With 5 ${}^5P_2 \times {}^6P_3 \times 4$ or ${}^5C_2 \times 2 \times {}^6C_3 \times 4! = 9600$ With A and 5 = ${}^4P_1 \times {}^6P_3 \times 8$or 4C1 × 2! × 6C3 × 4! × 8 = 3840	**M1**	One outcome correct, accept unsimplified.
	Required number = 6720 + 9600 – 3840	**M1**	Adding 'with a' to 'with 5' and subtracting 'A and 5'.
	12 480	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
5(c)	**Method 1 –** number of successful codes divided by total		
	$(1\times)\ 3\times {}^5\mathrm{P}_2$	**M1**	$3\times {}^5\mathrm{P}_n$, $n = 2, 3$. Condone $3\times {}^5\mathrm{C}_2$, no + or –.
	Probability = $\dfrac{their\ 3\times 5P2}{their\ 16\,800}$	**M1**	Probability = $\dfrac{their\ 60}{their\ 16\,800}$.
	$\dfrac{1}{280}$, 0.00357	**A1**	
	Method 2 – product of probabilities of each part of code		
	$\dfrac{1}{5}\times\dfrac{1}{4}\times\dfrac{1}{7}\times\dfrac{3}{6}\left(\times\dfrac{5}{5}\times\dfrac{4}{4}\right)$ or $\dfrac{1}{5}\times\dfrac{1}{4}\times\dfrac{3\times 5P2}{7P4}$	**M1**	$\dfrac{1}{5}\times\dfrac{1}{4}\times k$ where $0<k<1$ for considering letters.
		M1	$t\times\dfrac{1}{7}\times\dfrac{3}{6}$ or $t\times\dfrac{3\times 5P2}{7P4}$ where $0<t<1$.
	$\dfrac{1}{280}$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
6(a)	$p+q+0.65=1$	**B1**	Sum of probabilities = 1.
	$p+2q+0.15=0.55$	**B1**	Use given information.
	Solve 2 linear equations	**M1**	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p=0.3, \frac{3}{10}, \quad q=0.05, \frac{1}{20}$	**A1**	CAO, both WWW If **M0** with correct answers **SC B1**.
		4	
6(b)	Var (X) = $their\, 0.3+4\times their\, 0.05+9\times 0.05-0.55^2$	**M1**	Appropriate variance formula including $(\mathrm{E}(X))^2$, accept unsimplified.
	$0.6475\ \left[\frac{259}{400}\right]$	**A1**	CAO (must be exact).
		2	
6(c)	$1-\mathrm{P}(0,1,2)=1-({}^{12}\mathrm{C}_0\ 0.3^0\ 0.7^{12}+{}^{12}\mathrm{C}_1\ 0.3^1\ 0.7^{11}+{}^{12}\mathrm{C}_2\ 0.3^2\ 0.7^{10})$	**M1**	One correct term: ${}^{12}\mathrm{C}_x\, \mathrm{p}^x\, (1-\mathrm{p})^{12-x}$ for $0<\mathrm{x}<12$, $0<p<1$.
	$1-(0.01384+0.07118+0.16779)$	**A1FT**	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to **FT** *their p* from **6(a)** or correct.
	0.747	**A1**	
		3	
6(d)	$(0.95)^8\times 0.05\ =0.0332$ or $\ 0.95^8-0.95^9=0.0332$	**B1**	Evaluated.
		1	

Question	Answer	Marks	Guidance
7(a)	Probabilities: $\frac{x+1}{x+10}, \frac{9}{x+10}, \frac{x}{x+10}, \frac{10}{x+10}$	**B1**	One probability correct in correct position.
		B1	Another probability correct in correct position.
		B1	Other two probabilities correct in correct positions.
		3	
7(b)	$\frac{4}{10} \times \textit{their} \frac{10}{x+10}$	**M1**	Method consistent with *their* tree diagram.
	$\frac{4}{x+10}$	**A1**	AG
		2	

Question	Answer	Marks	Guidance
7(c)	$\frac{4}{x+10}=\frac{1}{6}$ $x+10=24, \quad x=14$	**B1**	Find value of x. Can be implied by correct probabilities in calculation.
	P(ARed|BRed) = P(ARed ∩ BRed) ÷ P(BRed) $\frac{\frac{6}{10}\times their\frac{x+1}{x+10}}{\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}}=\frac{\frac{6}{10}\times\frac{15}{24}}{\frac{6}{10}\times\frac{15}{24}+\frac{4}{10}\times\frac{14}{24}}=\frac{\frac{3}{8}}{\frac{73}{120}}$	**B1 FT**	$\frac{6}{10}\times their\frac{x+1}{x+10}$ as numerator or denominator of fraction.
		M1	$\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}$ seen anywhere.
		A1 FT	Seen as denominator of fraction.
	$\frac{45}{73}$, 0.616[4…]	**A1**	If B0 M0: **SC B1** for $\frac{\frac{3}{8}}{\frac{73}{120}}$ or $\frac{0.375}{0.6083}$ **SC B1** $\frac{45}{73}$ or 0.616.
		5	